Macroeconomics and the Term Structure[*]

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Abstract

This paper provides an overview of the analysis of the term structure of interest rates with a special emphasis on recent developments at the intersection of macroeconomics and finance. The topic is important to investors and also to policymakers, who wish to extract macroeconomic expectations from longer-term interest rates, and take actions to influence those rates. The simplest model of the term structure is the expectations hypothesis, which posits that long-term interest rates are expectations of future average short-term rates. In this paper, we show that many features of the configuration of interest rates are puzzling from the perspective of the expectations hypothesis. We review models that explain these anomalies using time-varying risk premia. Although the quest for the fundamental macroeconomic explanations of these risk premia is ongoing, inflation uncertainty seems to play a large role. Finally, while modern finance theory prices bonds and other assets in a single unified framework, we also consider an earlier approach based on segmented markets. Market segmentation seems important to understand the term structure of interest rates during the recent financial crisis.

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1 Introduction

On June 29, 2004, the day before the Federal Open Market Committee (FOMC) began its most recent tightening cycle, the overnight interest rate, the federal funds target, was one percent and the ten-year yield was 4.97 percent. On June 29, 2005, the corresponding rates were three percent and 4.07 percent. Over the course of a year when the Fed was tightening monetary policy, increasing the overnight rate by 2 percentage points, longer-term yields had instead fallen. The ten-year rate decreased by 90 basis points. Fixed mortgage rates and longer-term corporate bond yields fell even more. This rotation of the yield curve surprised then Fed Chairman Greenspan. In his oft-quoted February 2005 testimony to Congress, he stated:

> "This development contrasts with most experience, which suggests that ...increasing short-term interest rates are normally accompanied by a rise in longer-term yields... For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum."

But similar patterns in the configuration of interest rates have happened before—and since. Figure 1 shows the federal funds rate, three-month Treasury bill yields, and ten-year Treasury yields over the last seven years. The federal funds rate and three-month yields moved closely together, but ten-year (and other long-term) yields were often uncoupled from short-term rates. Greenspan's conundrum is one example. Another is that in the early fall of 2008, as the FOMC was cutting the federal funds rate sharply, long-term interest rates actually rose, peaking in early November of that year. This could be called the "conundrum in reverse." Later on, long-term yields declined sharply, around the time that the Fed announced the start of large-scale asset purchases.

The object of this paper is to discuss work on the macroeconomic forces that shape the term structure of interest rates. Broadly, the explanations fall into two categories. The first is that long-term interest rates reflect expectations of future short-term interest rates. This is the expectations hypothesis of the term structure of interest rates. If short-term interest rates are in turn driven by inflation and the output gap, as in the Taylor rule, then the term structure of interest rates ought to reflect expectations of future inflation and the output gap. For example, if the FOMC lowers policy rates today but, because of higher expected inflation, this leads agents to anticipate higher short-term interest rates in the future, then long-term interest rates could actually increase. The second category of explanations argues that long-term interest rates are also affected by risk premia, or by the effects of market segmentation, which can break the link between long-term interest rates and expectations of future short rates.

The literature on term structure modeling is vast. This paper portrays the state of that literature by presenting different theories in a unified framework. We look at which aspects of the data are explained by different models using term structure data from 1971 to the present, and discuss the macroeconomic foundations and implications of the different models. Our aim is to focus on interactions between macroeconomics, monetary policy, and the term structure, rather than to consider term structure models from a more technical finance perspective. Comprehensive reviews of the latter variety are already available in Duffie (2001), Singleton (2006) and Piazzesi (2008).

There are many reasons why policy-makers, investors and academic economists should and do care about the forces that affect the term structure of interest rates. First, economists routinely attempt to reverse-engineer market expectations of future interest rates, inflation, and other macroeconomic variables from the yield curve, but accomplishing this task also requires us to separate out any effects of risk premia. For example, in early 2010, the yield-curve slope was quite steep. Some commentators suggested that this steep yield curve represented concerns about a potential pickup in inflation, but without more formal models, it is hard to know if this was right, or if other forces were at work instead. Second, analysis of the term structure has implications for how monetary policy ought to respond to changes in long-term interest rates. If long-term rates were to fall because of an exogenous fall in risk premia, then it seems natural that policy-makers ought to *lean against the wind*[1] by tightening the stance of monetary policy to offset the additional stimulus to aggregate demand (McCallum (1994)). However the models that we shall discuss in this paper attempt to endogenize risk premia, and in this case the appropriate policy response is ambiguous and depends on the source of the change in risk premia (Rudebusch, Sack and Swanson (2007)). Third, at present, the federal funds rate is stuck at the zero bound. Monetary policy-makers may wish to provide additional stimulus to the economy. Under the expectations hypothesis, the only way that they can do this is by influencing market expectations of future monetary policy, perhaps by committing to keep the federal funds rate at zero for an extended period. On the other hand, if long-term interest rates are also buffeted by risk premia, then measures to alter those risk premia, perhaps through large-scale asset purchases, may be effective as well. The Federal Reserve and some other central banks have recently tried this. Fourthly, understanding the evolution of the term structure of rates is important for predicting asset returns and for determining the portfolio allocation choices of investors and their strategies for hedging interest rate risk. Finally, governments around the world borrow by issuing both short- and long-term debt, and debt that is

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[1] The whole term structure of interest rates should be relevant for aggregate demand. For example, business financing involves a mix of short-term commercial paper and long-term corporate bonds. In the U.S.—though not in foreign countries—most mortgages are fixed-rate.

both nominal and index-linked (inflation protected). Understanding the market pricing of these different instruments is important in helping governments determine the best mix of securities to issue in order to keep debt servicing costs low and predictable.

The plan for the remainder of this paper is as follows. Section 2 describes basic yield curve concepts and gives some empirical facts about the term structure of interest rates. Section 3 discusses the evidence on the expectations hypothesis of the term structure. Section 4 introduces affine term structure models, which the finance literature has been developing over the last ten years or so as a potential alternative to the expectations hypothesis. Progress has been rapid, and these models provide an alternative in which long-term interest rates represent both expectations of future short term interest rates and a time varying risk premium, or term premium, to compensate risk-averse investors for the risk of capital loss on selling a long-term bond before maturity and/or the risk of the bond's value being eroded by inflation. The models that are discussed span a spectrum from reduced form statistical models to fully specified structural dynamic stochastic general equilibrium (DSGE) models, and many intermediate cases. Section 5 examines the implications of structural breaks and learning for these models. Section 6 discusses term structure models with market segmentation, and section 7 concludes.

2 Basic Yield Curve Concepts and Stylized Facts

This section first introduces the basic bond pricing terminology that will be used in the remainder of the paper, and then presents the most salient stylized facts of the term structure of interest rates.

2.1 Basic yield curve concepts

The most basic building block of fixed income analysis is a default-risk-free zero-coupon bond. This security gives the holder the right to \$1 in nominal terms at maturity, with no risk of default.[2] Let $P_t(n)$ denote the price of an n-year zero-coupon bond at time t:

$$P_t(n) = \exp(-ny_t(n))$$

where $y_t(n)$ is the annualized continuously compounded yield on this bond. This bond pays the holder \$1 at time $t+n$, and we can solve for the yield from t to $t+n$ as:

$$y_t(n) = -\tfrac{1}{n}\ln(P_t(n))$$

At any point in time, bonds of different maturities will have different yields. A yield curve is a function that maps maturities into yields at a given point in time. Graphically, it is a plot of $y_t(n)$ against n. Figure 2 shows the yield curve out to ten years in the first and last months of our sample, as well as the average yield curves (i.e. the yields at each maturity averaged over the sample period). As is clear from the figure, a stylized fact is that the yield curve slopes up on average. This has important repercussions for reverse engineering the yield curve to obtain expectations and term premia.

It is often more instructive to analyze long-term yields in terms of their constituent forward rates. The two-year yield observed today can be thought of as a one-year contract, with a commitment to roll over at a rate specified today at the end of the first year. Since we observe the both the one-year and two-year yields, it should be possible to infer the rate implicitly agreed on today for the second year. This is a one-year-ahead one-year forward rate. More generally,

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[2] We think of government bonds as being for all practical purposes free of nominal default risk but for some countries even sovereign debt may require modeling of default risk as well. And of course, the value of any nominal bond is always at risk of being eroded by inflation.

a forward rate is the yield that an investor would require today to make an investment over a specified period in the future—for m years beginning n years hence. The continuously compounded return on that investment is the m-year forward rate beginning n years hence and is given by:

$$f_t(n,m) = -\frac{1}{m}\ln(\frac{P_t(n+m)}{P_t(n)}) = \frac{1}{m}((n+m)y_t(n+m) - ny_t(n)) \tag{1}$$

Taking the limit of (1) as m goes to zero gives the instantaneous forward rate n years ahead, which represents the instantaneous return for a future date that an investor would demand today:

$$\lim_{m\to 0} f_t(n,m) = f_t(n,0) = y_t(n) + n\frac{\partial y_t(n)}{\partial n} = \frac{\partial n y_t(n)}{\partial n} = -\frac{\partial}{\partial n}\ln(P_t(n)) \tag{2}$$

One can think of a zero-coupon bond as a string of forward rate agreements over the horizon of the investment, and the yield therefore has to equal the average of those forward rates. Specifically, from (2) we can write

$$y_t(n) = \tfrac{1}{n}\Sigma_{i=1}^{n} f_t(i-1,1) = \tfrac{1}{n}\int_0^n f_t(s,0)ds$$

The beauty of forward rates is that they allow us to isolate long-term determinants of bond yields that are separate from the mechanical effects of short-term interest rates.

Figure 3 shows a long time series of three-month and ten-year yields, along with ten-year-ahead instantaneous forward rate in the U.S. Yields and forward rates generally drifted higher over the 1970s and then reversed course over the last thirty years, following the general pattern of inflation and longer-run inflation expectations. But there was also much variation associated with the business cycle. Short-term interest rates were highly procyclical, as the FOMC sought to alter the stance of monetary policy to limit cyclical fluctuations in inflation and output. On the other hand, forward rates were, if anything, countercyclical.

In Figure 3, the usefulness of forward rates as an analytical device is evident at the end of 2009. Ten-year yields were at unusually low levels, by historical standards. However, long-term forward rates were somewhat above their average level over the previous decade; the unusually low level of long-term yields was solely the mechanical effect of short-term interest rates being low, as the FOMC had set the federal funds rate to zero and expressed the intention of keeping it there for an extended period.

Another illustration of the usefulness of forward rates comes in looking at the effects of macroeconomic news announcements on yields. Naturally, announcements of stronger-than-expected economic data cause interest rates to increase, as they presage a tighter stance of monetary policy. However, a more detailed analysis can be obtained by looking at the effects of these announcements on the term structure of forward rates. Gürkaynak, Sack and Swanson (2005) find that stronger-than-expected economic data leads even ten-year-ahead forward rates to jump higher. This seems very unlikely to owe to any information about the state of the business cycle. A possible interpretation, proposed in that paper, is that long-term inflation expectations are poorly anchored. We return to this and alternative interpretations of the behavior of forward rates in section 4.

Another essential tool of term structure analysis is the *holding period return*. The holding period return is the return on buying an n-year zero-coupon bond at time t and then selling it, as an $(n-m)$-year zero-coupon bond, at time $t+m$. This return is

$$hpr_t(n,m) = \tfrac{1}{m}\left[\ln(P_{t+m}(n-m) - \ln(P_t(n))\right]$$

and the difference between this and the m-year yield is the *excess holding period return*:

$$exr_t(n,m) = hpr_t(n,m) - y_t(m)$$

Figure 4 shows the excess holding period returns of the ten-year over one-year bonds over the sample period. These are on average positive—which follows from the average upward slope of the yield curve, shown in Figure 2—and also tend to be especially high at the beginning of recoveries from recessions. This is an important feature of the data that term structure models have to match.

2.2 The expectations hypothesis

The Expectations Hypothesis (EH) is the benchmark term structure model. In its strong form, it asserts that long-term yields are equal to the average of expected short-term interest rates until the maturity date. In its weak form, it allows for a constant term premium of the long yield over the average expected short-term interest rate. That term premium may be maturity-specific but does not change over time.

More formally, in its strong form, the EH states that investors price all bonds as though they were risk-neutral. That is, investors care only about expected outcomes (means of probability distributions), and will be indifferent between two assets with the same expected return but different levels of uncertainty. This implies that the price of an n-year zero-coupon bond is:

$$P_t(n) = E_t(\exp(-\int_0^n r(t+s)ds)) \tag{3}$$

where $r(t) = y_t(0)$ is the instantaneous risk-free interest rate. Taking the logs of both sides and neglecting a Jensen's inequality term gives:

$$y_t(n) \simeq \tfrac{1}{n} E_t(\int_0^n r(t+s)ds)$$

That is, the long-term interest rate is the average expected future short-term interest rate over the life of the bond. The Jensen's inequality term—arising because the log of an expectation is not the same as the expectation of a log—will tend to push long-term yields down, below the average of expected future short-term interest rates. This is the reason why at very long maturities (of about 20 years and longer), the yield curve typically slopes down. However, at maturities of about ten years or less, the Jensen's inequality effect is modest. For this reason, we neglect it henceforth in this paper as is customary in the literature.

Equivalently, in its strong form, the EH implies that instantaneous forward rates are equal to expectations of future short term interest rates:

$$f_t(n,0) = E_t(r(t+n))$$

and that expected excess holding period returns are zero:

$$E_t(exr_t(n,m)) = 0$$

The yield curve that would be realized with rational agents in the absence of arbitrage under risk neutrality is described by the expectations hypothesis, making it the natural benchmark for the study of the term structure of interest rates.

2.3 Risk premia and the pricing kernel

Economists generally believe that agents are risk-averse (see, for example, Friedman and Savage (1948)). However, even under risk aversion, the pricing of different contingent cash flows has to be internally consistent to avoid arbitrage opportunities. More precisely, the absence of arbitrage implies that there exists a strictly positive random variable, M_{t+1}, called the *stochastic discount factor* or the *pricing kernel*, such that the price of any asset at time t obeys the pricing relation:

$$P_t = E_t(M_{t+1}P_{t+1}) \tag{4}$$

where the price at time $t+1$ includes any dividend or coupon payment that has been received. The stochastic discount factor is the extension of the ordinary discount factor to an environment with uncertainty and possibly risk-averse agents (see Hansen and Renault, 2009, for a detailed discussion of pricing kernels). Since the payoff of an n-year zero-coupon bond is deterministic and is equal to \$1 at maturity, equation (4) implies that in period $t+n-1$, when the security has one year left to maturity, its price will be

$$P_{t+n-1}(1) = E_{t+n-1}(M_{t+n})$$

Iterating this backwards and using the law of iterated expectations, the bond price today will be

$$P_t(n) = E_t(\textstyle\prod_{i=1}^{n} M_{t+i}) \qquad (5)$$

Equation (5) makes no assumption of risk-neutrality and so does not imply that the EH holds. If risk-neutrality were to hold, then $M_{t+1} = E_t(\exp(-\int_0^1 r(t+s)ds))$ and so equation (5) would collapse to equation (3), and long-term yields would be equal to the expected average future short-term interest rate as is the case under EH. But since we make no assumption of risk-neutrality, there may be a gap between long-term yields and the average expected future short-term interest rate. This is called the risk premium, or term premium:

$$rp_t(n) = y_t(n) - n^{-1}E_t(\textstyle\sum_{i=0}^{n-1} y_{t+i}(1)) \qquad (6)$$

that compensates risk-averse investors for the possibility of capital loss on a long-term bond if it is sold before maturity and/or the risk of the bond's value being eroded by inflation.[3]

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[3] Although the payoff of a bond at maturity is known with certainty, the value of a long-term bond before maturity is uncertain. That is, the resale value of the bond before maturity (or the opportunity cost of funding the bond position) depends on the uncertain trajectory of future short term interest rates.

Equation (6) is effectively an "accounting" definition of the risk premium—by construction, any change in long-term yields that is not accompanied by a corresponding shift in expectations of future short-term interest rates must result in a change in the risk premium. This could be a change in the risk premium from an asset pricing model (as will be considered in section 4), or it could result from the effects of market segmentation (as discussed in section 6—a setup in which equations (4) and (5) do not apply). Any gap between yields and actual expectations is always defined as the risk (or term) premium.

2.4 Index-linked bonds

About thirty years ago, the United Kingdom started issuing index-linked bonds—government bonds with principal and coupons that are tied to the level of the consumer price index.[4] These securities compensate the holder for the accrued inflation from the time of issuance date to the time of payment date for each cash flow date. The United States began the Treasury Inflation Protected Securities (TIPS) program in 1997, and many countries now offer index-linked debt to investors. Gürkaynak, Sack and Wright (2010) provide detailed information on the TIPS market.

The spread between nominal and indexed yields provides information on investors' perceptions of future inflation, known as *breakeven inflation*[5] or *inflation compensation*. Thus, the existence of inflation-indexed bonds has helped relate the nominal term structure to macroeconomic fundamentals by allowing for a decomposition of nominal yields into real and inflation-related components. But, just as investors' pricing of nominal bonds may be distorted by risk premia, the same is true for the pricing of index-linked bonds, and so both the real

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[4] This was the first large-scale modern index-linked government bond market, although there is a centuries-long history of bonds that include some form of protection against inflation, such as being denominated in gold or silver.

[5] This spread is called breakeven inflation because it is the rate of inflation that, if realized, would leave an investor indifferent between holding a nominal or a TIPS security.

rates and breakeven inflation rates may be affected by risk premia. We return to discuss these issues further in section 4 below.

3 Testing the Expectations Hypothesis

The expectations hypothesis is a natural starting point to study the term structure of interest rates and also to relate macroeconomic fundamentals to the yield curve. Indeed, if the EH were sufficient to explain the term structure, then expected short rates could be directly read from the yield curve. However, the fact that yield curves normally slope up is at odds with the simple EH because without term premia this would have to imply that short-term interest rates are expected to trend upwards indefinitely. Therefore, the relevant form of the EH must be the weak from, which allows maturity specific term premia that are constant over time. This is how we define the "expectations hypothesis" for the remainder of the paper.

Given its assumption of constant term premia, the EH attributes all *changes* in the yield curve to *changes* in expected short rates. As an accounting matter, the EH would imply that the $1\frac{3}{4}$ percentage point decline in long-term forward rates from June 2004 to June 2005 must represent a fall of this magnitude in long-term expectations of inflation and/or the real short-term interest rate. It would also imply that the rebounds in forward rates during the early fall of 2008 and again in late 2009 represent increases in long-term expectations of inflation and/or real rates. Thus, under the EH, changes in the term structure can be used to infer changes in investors' expectations concerning the path of monetary policy. If, in addition, the central bank's rule relating monetary policy to macroeconomic conditions were known by those investors, then we could also read off changes in their expectations of the state of the economy.

In this section, we present evidence from some well-known tests of the EH and point out some anomalies in the term structure, from the viewpoint of the EH, beginning with a very influential approach proposed by Campbell and Shiller (1991). They proposed two tests which both test the implication of the EH that when the yield curve is steeper than usual, both short and long term rates must be expected to rise.[6] Conversely, if the yield curve is flatter than usual, short- and long-term rates must be expected to fall.

The first Campbell and Shiller (1991) test is based on the implication of the EH that the n-period interest rate is the expected average m-period interest rate over the next $k = n/m$ intervals each of length m. That is, the return on lending for n periods today and the expected average return on lending for m periods and rolling over $k - 1$ times should be equal:

$$y_t(n) = \tfrac{1}{k} E_t(\Sigma_{i=0}^{k-1} y_{t+im}(m))$$

neglecting a constant. This means that

$$\begin{aligned} y_t(n) - y_t(m) &= \tfrac{1}{k} E_t(\Sigma_{i=0}^{k-1} y_{t+im}(m)) - y_t(m) \\ \therefore y_t(n) - y_t(m) &= \Sigma_{i=1}^{k-1}(1 - \tfrac{i}{k}) E_t(y_{t+im}(m) - y_{t+(i-1)m}(m)) \end{aligned}$$

and so if we consider the regression

$$\Sigma_{i=1}^{k-1}(1 - \frac{i}{k})(y_{t+im}(m) - y_{t+(i-1)m}(m)) = \alpha + \beta(y_t(n) - y_t(m)) + \varepsilon_t \qquad (7)$$

which is a regression of a weighted-average of future short-term yield changes onto the slope of the term structure, then one ought to get a slope coefficient β that is equal to one. The dependent variable in equation (7) can be thought of as the *perfect-foresight* term spread, as it is the term spread that would prevail at time t if the path of period interest rates over the next m periods were correctly anticipated.

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[6] Long rates as well as short rates are expected to increase when the yield curve is steep (under the EH) because with a steep yield curve distant-horizon forward rates are higher than short-term forward rates.

In Table 1, we report the results of the estimation of equation (7) using end-of-month U.S. yield curve data from the dataset of Gürkaynak, Sack and Wright (2007) from August 1971 to December 2009 for different choices of m and n. Newey-West standard errors with a lag truncation parameter of m are used, because the overlapping errors will induce a moving average structure in ε_t. Like Campbell and Shiller (1991), we find that the point estimates of the slope coefficient are all positive, but less than one. Some, but not all, are significantly different from one. Overall this test gives only weak evidence against the EH.

The second Campbell and Shiller (1991) test is based on the implication of the EH that the expectation of the future interest rate from m to n periods hence is the forward rate over that period (again neglecting a constant). So

$$\begin{gathered} E_t(y_{t+m}(n-m)) = \tfrac{n}{n-m} y_t(n) - \tfrac{m}{n-m} y_t(m) \\ \therefore E_t(y_{t+m}(n-m) - y_t(n)) = \tfrac{m}{n-m}(y_t(n) - y_t(m)) \end{gathered}$$

and, in the regression

$$y_{t+m}(n-m) - y_t(n)) = \alpha + \beta[\frac{m}{n-m}(y_t(n) - y_t(m))] + \varepsilon_t \tag{8}$$

which is a regression of the change in long-term yields onto the slope of the term structure, the slope coefficient β should again be equal to one.

In Table 2, we report the results of the estimation of equation (8). Like Campbell and Shiller, we find that the estimates of β are all negative and significantly different from one, and become more negative as n increases. When the yield curve is steep, according to the EH, long-term interest rates should subsequently *rise*, but in fact they are more likely to *fall*. This term structure anomaly has been known for a long time, going back to MacAulay (1938). It is closely related to the finding of Shiller (1979) that long-term yields are too volatile to be rational expectations of average future short-term interest rates.

Another, related, approach to testing the EH was considered by Fama and Bliss (1987), Backus, Foresi, Mozumdar and Wu (2001), Duffee (2002) and Cochrane and Piazzesi (2005, 2008). This involves regressing the excess returns on holding an n-year bond for a holding period of m years over the return on holding an m-year bond for that same period onto the term structure of interest rates at the start of the holding period. Under the EH, term premia are time-invariant, and so *ex-ante* expected excess returns should be constant, and all of the coefficients on the right-hand-side variables should jointly be equal to zero.

For example, following Cochrane and Piazzesi (2008), one could regress excess returns on holding a five-year bond for one year over the return on holding a one-year bond onto one-year forward rates ending one, three, and five years hence, estimating the regression:

$$exr_t(n, 1) = \beta_0 + \beta_1 y_t(1) + \beta_2 f_t(2, 1) + \beta_3 f_t(4, 1) + \varepsilon_t \tag{9}$$

This is a regression of the excess returns that are realized over the year on observed forward rates at the beginning of the year. The EH predicts that the slope coefficients should all be equal to zero. The coefficients from estimating equation (9) over the sample period from August 1971 to December 2009 are shown in Table 3. Again the EH is rejected. According to the EH, none of the forward-rates on the right-hand-side should have any predictive power for excess returns. But the R^2 values for this regression range from 12 to 20 percent. Table 3 also shows the results from estimating this regression over a period that excludes the recent financial crisis (August 1971 to December 2006). For this earlier period, the rejection of the EH is even more decisive.

There is thus a good bit of evidence of anomalies in the term structure that the EH cannot account for. But a number of caveats should be pointed out with this assessment. First, there are econometric issues associated with estimating equations (8) and (9) with relatively short spans of data. Both are regressions relating quite persistent variables, and ordinary distribution theory often provides a poor guide to the small sample properties of estimators and test statistics under these circumstances. It's a bit like running a regression of one trending variable on another, which has the well-known potential to result in a spurious regression. Also, the regressions are subject to the possibility of peso problems in which yields are priced allowing for the possibility of a regime shift that was not actually observed in the short sample. Bekaert and Hodrick (2001) and Bekaert, Hodrick and Marshall (2001) both consider the two tests of Campbell and Shiller (1991), but provide alternative critical values that are more appropriate in small samples, given these problems. Even with these adjustments, they continue to reject the EH, although less strongly.

Second, some authors have examined evidence on the expectations hypothesis for very short maturity bonds and obtained mixed results. Rudebusch (1995) and Longstaff (2000) considered regressions of the form of equations (8) and (9) where the maturity of the "long bond" is measured in days or weeks. Little evidence is found against the EH. However, Piazzesi and Swanson (2008) conducted a similar exercise with short-term federal funds futures, and rejected the EH.

Third, Froot (1989) considered a different approach to testing the expectations hypothesis. He compared forward rates with survey-based expectations of future interest rates. For short-term rates, the two diverged, indicating a failure of the EH. But for long-term rates, Froot found that the survey-based and forward rates agreed quite closely. The flipside of this is that the errors in survey forecasts for interest rates seem to be quite easy to predict ahead of time, suggesting that the survey forecasts may not be fully rational (Bachetta, Mertens and van Wincoop (2010)). But it is consistent with the apparent failure of the EH being in part due to agents' learning about structural changes in the economy.

Finally, most empirical work finding problems with the expectations hypothesis has been conducted using post-war U.S. data. Authors considering earlier sample periods or other countries have obtained more mixed results. For example, Hardouvelis (1994) estimated equation (8) for all the G7 countries, and found that the evidence against the EH was much weaker for countries other than for the U.S.[7] Mankiw and Miron (1986) estimated equation (7) over sample periods from before the foundation of the Federal Reserve system in 1914 and found support for the EH. Overall, the sample periods or countries for which the EH finds most support are ones during which long-run inflation expectations were presumably well anchored, such as the U.S. under the gold standard or countries such as Germany and Switzerland that held inflation in check even in the late 1970s. And the cases where the EH fares relatively poorly are ones with heightened inflation uncertainty and/or ones in which the central bank smoothed interest rates so that they are well approximated by a random walk specification.

Overall there appear to be a number of features of the term structure of interest rates that the EH has trouble explaining. The standard finance explanation is that this is due to time-variation in risk premia. In the next section, we turn to models with time-varying risk premia and ask what information about macroeconomic fundamentals can be uncovered by separating expected short rates from time-varying term premia. But the anomalies could owe in part to changes in long-run inflation expectations about which agents learn slowly. Accordingly, we consider learning and structural change in section 5. We discuss an approach advocated by Kozicki and Tinsley (2005) in which long-term

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[7] Other authors finding more support for the EH when applied to foreign countries include Gerlach and Smets (1997), Jondeau and Ricart (1999), Bekaert, Hodrick and Marshall (2001) and Bekaert, Wei and Xing (2007).

interest rates are given by agents' beliefs about average expected future short rates—and so the EH holds after all—but where these beliefs are conditioned on agents' perceptions of the central bank's long-run inflation target, not the true inflation target. The agents' perceptions of the long-run inflation target are in turn formed by backward looking adaptive expectations. Kozicki and Tinsley argue that this model can explain many stylized facts of the term structure. Finally, the configuration of interest rates could reflect some market segmentation, a possibility that has generally been overlooked in the macro-finance literature, but which we will consider in section 6. We argue that this approach may be helpful for understanding the behavior of long-term interest rates at times of unusual market turmoil, such as during the recent financial crisis.

We end this section by noting that researchers are now beginning to have enough data to obtain empirical evidence on the pricing of index-linked bonds. Evans (1997) and Barr and Campbell (1997) have applied tests of the EH to index-linked bonds in the U.K., with mixed results. Only a shorter span of data on inflation-protected bonds is available for the U.S., but with the available data it is striking how closely the long-term nominal and index-linked bond term structures track each other. Figure 5 shows the TIPS and nominal ten-year-ahead instantaneous forward rates. As can be seen in Figure 5, these two forward rates have moved almost in lockstep over the past ten years (see also Campbell, Shiller and Viceira (2009)).[8] The TIPS market is still young and less liquid than the nominal Treasury market, but this observation appears to suggest that a complete model of nominal term structure patterns will have to take account of real rate risk, as well as inflation risk.

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[8] In other words, long-term forward breakeven inflation rates have been far more stable than long-term forward real rates.

4 Affine Term Structure Models

Affine term structure models provide an alternative to the expectations hypothesis.[9] They have become enormously popular in the finance literature in the last ten years. A natural approach to term structure analysis would be to forecast interest rates at different maturities in a vector autoregression (VAR). Yields today are helpful for forecasting future yields (Campbell and Shiller (1991), Diebold and Li (2006) and Cochrane and Piazzesi (2005)), so this should be a viable approach to understanding how interest rates move over time. The trouble with this is that using the estimated VAR can—and typically will—imply that there is some clever way that investors can combine bonds of different maturities to form a portfolio that represents an arbitrage opportunity: positive returns without any risk. If we don't believe that investors leave twenty dollar bills on the sidewalk, then it is important to exploit the predictability of future interest rates (from the VAR) in a framework that rules out the possibility of pure arbitrage. This is what affine term structure models do.

In this section we will lay out affine models with progressively more economic structure that will allow us not only to represent term premia statistically, but also to understand the economic forces at work. We will argue that the hedging of inflation risk is an important driver of bond risk premia. We will conclude the section with a short discussion of how index-linked bonds can be incorporated into the affine model framework.

The basic elements of a standard affine term structure model are as follows:

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[9] Affine models are models in which yields at all maturities are "affine" (linear plus a constant) functions of one or more factors. Most of the models discussed in this section are affine, but strictly speaking a few are models in which yields are instead nonlinear functions of the factors. While "factor-based term structure models" would have been a more precise section title, most of the models considered here are typically referred to as "affine" models. We thought it would be more helpful to introduce them as such.

(a) There is a kx1 vector of (observed or latent) factors that follows a VAR:

$$X_{t+1} = \mu + \Phi X_t + \Sigma \varepsilon_{t+1} \tag{10}$$

where is ε_t iid $N(0, I)$.

(b) The short-term interest rate is an "affine" (linear plus a constant) function of the factors:[10]

$$y_t(1) = \delta_0 + \delta_1' X_t \tag{11}$$

(c) The pricing kernel is conditionally lognormal

$$M_{t+1} = \exp(-y_t(1) - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}) \tag{12}$$

where $\lambda_t = \lambda_0 + \lambda_1 X_t$. Thus the set of factors that determine the short rate also determine the long rates through the pricing kernel.

Langetieg (1980) showed that equations (5), (10), (11) and (12) imply that the price of an n-period zero-coupon bond is

$$P_t(n) = \exp(A_n + B_n' X_t) \tag{13}$$

where A_n is a scalar and B_n is a $kx1$ vector that together satisfy the recursions

$$A_{n+1} = -\delta_0 + A_n + B_n'(\mu - \Sigma\lambda_0) \tag{14}$$

$$B_{n+1} = (\Phi - \Sigma\lambda_1)' B_n - \delta_1 \tag{15}$$

starting from $A_1 = -\delta_0$ and $B_1 = -\delta_1$. Zero-coupon yields are accordingly

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[10] This model does not impose the zero-bound on interest rates. Kim (2008) discusses some extensions that do impose the zero bound.

given by

$$y_t(n) = -\frac{A_n}{n} - \frac{B_n'}{n} X_t \tag{16}$$

This model is called an "affine" model, because yields at all maturities are all affine functions of the factors. Although other assumptions on the functional form of the pricing kernel and short-term interest rate are of course possible, the affine model is most popular in part because of its tractability.

If $\lambda_0 = \lambda_1 = 0$, then equations (5) and (11) imply that investors are risk-neutral and the strong-form expectations hypothesis holds: $P_t(n) = E_t \exp(-\Sigma_{i=0}^{n-1} y_{t+i}(1))$. But we do not impose this restriction. The bond prices in equation (13) are however the same *as if* agents were risk-neutral but the vector of factors followed the law of motion

$$X_{t+1} = \mu^* + \Phi^* X_t + \Sigma \varepsilon_{t+1} \tag{17}$$

where $\mu^* = \mu - \Sigma\lambda_0$ and $\Phi^* = \Phi - \Sigma\lambda_1$ instead of equation (10). Equations (10) and (17) are known as the *physical* and *risk-neutral* laws of motion for the factors, or P and Q measures, respectively. Intuitively, the risk-neutral law of motion uses a distorted data generating process, overweighting states of the world in which investors' marginal utility is high.

Many papers have estimated models of the form of equations (10) - (17). One very common approach is to infer the factors X_t from the current cross-section of interest rates—the factors are either yields, or they are unobserved latent variables (see for example Duffie and Kan (1996), Dai and Singleton (2000, 2002), Duffee (2002), Kim and Orphanides (2005) and Kim and Wright (2005)). As three principal components are sufficient to account for nearly all of the cross-sectional variation in bond yields (Litterman and Scheinkman (1991)), most of these papers use three yield-curve factors in X_t, which can be interpreted as the level, slope, and curvature of yields. Christensen, Diebold and Rudebusch

(2007) consider an affine term structure model with three latent factors in which μ and Φ are unrestricted, but $\mu^* = 0$ and

$$\Phi^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\tau & \tau \\ 0 & 0 & 1-\tau \end{pmatrix}$$

where τ is a parameter. Under these restrictions, equation (16) reduces to

$$y_t(n) \simeq X_{1t} + X_{2t}\frac{1-\exp(-n/\tau)}{n/\tau} + X_{3t}[\frac{1-\exp(-n/\tau)}{n/\tau} - \exp(-n/\tau)] \quad (18)$$

where $X_t = (X_{1t}, X_{2t}, X_{3t})'$ is the state vector.[11] This model has the appealing feature that the yields follow the functional form of Nelson and Siegel (1987) that has been found to fit yield curves quite well—the elements of the state vector are just Nelson and Siegel's level, slope, and curvature measures.

Term structure models with latent factors can be estimated by maximum likelihood using the Kalman filter as in the model of Christensen, Diebold and Rudebusch (2007). Figure 6 shows estimates of ten-year term premia in the U.S. from this model.[12] The term premium estimates rose in the 1970s, but then trended lower from about 1985 to 2000. They tend to be countercyclical—higher in recessions than in expansions (Fama (1990) and Backus and Wright (2007)). Also, term premium estimates fell to the lowest levels in the sample in 2004 and 2005, offering at least a partial explanation of Greenspan's conundrum. Different models of course produce different estimates of term premia, but many of them agree on these points. Rudebusch, Sack and Swanson (2007) compare

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[11] The model of Christensen, Diebold and Rudebusch is written in continuous time: here we are writing the discrete time representation of the law of motion of the state vector under the risk-neutral measure. Also note that equation (18) is an approximation, because it omits a remainder term that is time-invariant, and depends just on the bond maturity, n.

[12] We implement estimation of this model using end-of-quarter data on yields at maturities of 3 months, 6 months and 1, 2,....10 years. These yields are all assumed to be given by equation (18) plus iid N(0,σ^2_{ME}) measurement error. We specify that Σ is a diagonal variance-covariance matrix. The parameters of the model are thus μ, Φ, τ, σ^2_{ME} and the diagonal elements of Σ.

five different term premium estimates and find that they all agree on some key points, particularly the downward trend in bond risk premia over the 1990s. We will return to the interpretation of this downward trend later. Judging from the Christensen, Diebold and Rudebusch model, term premia rose in 2009, although remained low by historical standards.

Approaches with either latent variables or yields as factors have the advantage of providing a close fit to observed interest rates using a small number of variables. But they have the drawback that they lack economic interpretation. It would be hard to tell a policymaker that the key to having lower and more stable risk premia is to change the law of motion of some latent factor. The remainder of this section moves incrementally towards models with more economic structure.

4.1 Term structure models with macroeconomic factors

Some authors use macroeconomic variables as factors instead. Bernanke, Reinhart and Sack (2004) use an affine model given by equations (10) - (17) in which the factors are GDP growth, inflation, the federal funds rate, and survey-expectations of future inflation and growth. Similarly, in Smith and Taylor (2009), the factors are inflation and the output gap. This means that short-term interest rates depend on inflation, π_t, and the output gap, gap_t:

$$y_t(1) = \delta_0 + \delta_{1,1}\pi_t + \delta_{1,2}gap_t$$

Equation (16) then implies that yields at all maturities are affine functions of current inflation and the output gap:

$$y_t(n) = a_0(n) + a_1(n)\pi_t + a_2(n)gap_t$$

Smith and Taylor use the model to interpret yield curve movements. For example, they propose an interpretation of Greenspan's conundrum, in which it owes to the Fed being perceived to have lowered the sensitivity to inflation, $\delta_{1,1}$, in its Taylor rule. This caused the whole term structure of inflation response coefficients, $a_1(n)$ to move lower, and long-term yields declined, even as short-term interest rates climbed.

Models with macroeconomic variables as factors allow the response of the yield curve to macroeconomic shocks to be analyzed. However, they do not fit observed yields quite as well as latent factor models. A possible approach is to combine both macroeconomic and latent variables as factors. Ang and Piazzesi (2003) provide a model in this category. They consider using as factors the first principal component of a set of inflation measures, the first principal component of a set of measures of real economic activity, and three latent factors. In the equation for the short-term interest rate (equation (11)), Ang and Piazzesi restrict the short rate to depend on inflation and economic activity alone, in as in the Taylor rule.

The inclusion of macroeconomic variables as factors raises two issues. Firstly, Ang and Piazzesi (2003) restrict the VAR in equation (10) so that the yield curve factors have no effects on future inflation or output. Similar restrictions are imposed by Hördahl, Tristani and Vestin (2006). The propagation of shocks is thus uni-directional. That seems a strong restriction, which in turn raises the question of why the central bank would want to adjust interest rates to influence the macroeconomy. More recent papers have allowed for feedback between macroeconomic variables and yields. Diebold, Rudebusch and Aruoba (2006) consider a model with both yield curve and macroeconomic factors in which the VAR in equation (10) is unrestricted. Empirically, they find that yields affect future values of the macroeconomic variables, and vice-versa. Nimark (2008) finds that central banks using the information in yields about macroeconomic fundamentals can improve welfare.

There is a second and more thorny issue with the use of macroeconomic variables in affine models. Equation (16) relates the yield on an n-period bond to the factors. Using this equation for a set of different maturities gives a system of equations that one ought normally be able to use to solve for the factors from the observed yields. Thus, if macroeconomic variables are truly factors, then a regression of these variables onto yields ought to give a very good fit. However, regressing macroeconomic variables on yields consistently gives small to moderate R^2 values. This point is made by Rudebusch and Wu (2008), Joslin, Priebsch and Singleton (2009), Kim (2009), Orphanides and Wei (2010) and Ludvigson and Ng (2009). A way around this—proposed by Joslin, Priebsch and Singleton, Ludvigson and Ng, and Rudebusch and Wu—is to consider models in which knife-edge parameter restrictions are satisfied, such that yields of all maturities have a loading of zero on the macroeconomic variables in equation (16). This means that there is a singularity whereby one cannot invert equation (16) to recover the macroeconomic variables from yields. This does not prevent yields from having forecasting power for future values of the macroeconomic variables. Changes in macro variables can affect future yield curves and expectations of future short-term interest rates, but they have an offsetting impact on term premia. The two effects cancel out, leaving today's term structure unchanged. The terminology used to describe this situation is that macroeconomic variables are *unspanned* factors.[13]

4.2 Structural models of factor dynamics

The term structure models considered up to this point use an unrestricted VAR in equation (10) to model the dynamics of the factors. And the stochastic discount factor is likewise driven by factors in an atheoretical way, given in

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[13] Macroeconomic variables are not the only possible candidates for unspanned factors. Collin-Dufresne and Goldstein (2002) and Andersen and Benzoni (2008) argue that bond derivatives contain a factor that is not reflected in the term structure of yields.

equation (12). More structural approaches are however available in which the law of motion of the factors, or the stochastic discount factor, or both, are grounded in some economic model based on utility maximization.

This subsection considers models with the stochastic discount factor given by equation (12), but in which economic theory is used to motivate the law of motion of the factors. The economic theory could be a new-Keynesian macroeconomic model, that in turn has microeconomic foundations. In this setup rather than an unrestricted VAR, the macroeconomic factors are driven by the model dynamics. Inflation depends on expected future inflation, past inflation, and the output gap, in the hybrid new-Keynesian Phillips curve. Meanwhile, in the IS equation, the output gap depends on expectations of the future output gap, the past output gap, and the real short-term interest rate. Rudebusch and Wu (2007) is a model of this sort. The equations describing the evolution of these macroeconomic factors can be written as forward-looking linear difference equations with rational expectations. Solution techniques for these equations have been proposed by a number of authors including Blanchard and Kahn (1980) and Sims (2001). The solution implies that the macro variables follow a restricted vector autoregression, that can however still be written in the form of equation (10). Other models in this family include Gallmeyer, Hollifield, and Zin (2005) and Rudebusch, Swanson, and Wu (2006).

These models are better able to offer explanations grounded in economic theory for yield curve movements, as the driving factors are now restricted to behave in a model-consistent manner. However, the key ingredient of the model—the pricing kernel that maps the factors into yields—remains *ad hoc.* We now turn to models with pricing kernels that are based on utility maximization.

4.3 Risk premia from utility maximization

The models considered in this section so far are all able to match the empirical properties of the yield curve reasonably well. They get the slope of the yield curve right, and they match the anomalies documented by Campbell and Shiller (1991) and others. But they are based on a statistical model for the pricing kernel. That is, equation (12) is a reduced form expression for the pricing kernel that generates reasonable and tractable results, but the pricing kernel and the utility maximization that takes place in the macroeconomic model may not be consistent with each other. In this subsection, we now turn to discussing papers that have instead derived the pricing kernel from an explicit utility maximization problem, while going back to having unrestricted reduced form dynamics for the factors.

The first papers to analyze the term structure of interest rates with a structural model of the pricing kernel had great difficulty in matching the most basic empirical properties of yield curves—notably that yield curves on average slope up indicating that nominal bond risk premia are typically positive. For example, Campbell (1986) considered an endowment economy in which consumption follows an exogenous time series process and a representative agent trades bonds of different maturities and maximizes the power (or constant relative risk aversion) utility function

$$E_t \Sigma_{j=0}^{\infty} \frac{\beta^j c_{t+j}^{1-\gamma}}{1-\gamma} \tag{19}$$

where c_t denotes consumption at time t, β is the discount factor and γ is the coefficient of relative risk aversion. The pricing kernel is therefore $M_{t+1} = \beta \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}}$ which is the ratio of marginal utility tomorrow to marginal utility today. The term premium on bonds in this economy depends on the nature of the consumption process. If the exogenous consumption growth process is positively autocorrelated, then risk premia on long-term bonds should be negative.

The intuition is that expected future consumption growth falls, and bond prices rise, in precisely the state of the world in which marginal utility is high. The long-term bond is therefore a good hedge, and the risk premium is negative. Therefore a positively autocorrelated consumption growth process would generate negative risk premia. Conversely, a negatively autocorrelated consumption growth process would generate positive risk premia.

The problem with this story is however that consumption is close to being a random walk, implying that term premia should be close to zero. Thus these standard consumption-based explanations are hard to reconcile with the basic fact that yield curves ordinarily slope up. Backus, Gregory and Zin (1989) likewise discussed the difficulty of consumption based asset pricing models in matching the sign, magnitude and other properties of bond risk premia. Donaldson, Johnson and Mehra (1990) and Den Haan (1995) were also unable to match the sign and magnitude of bond risk premia in real business cycle models.[14] Intuitively, the problem is that we generally think of recessions—periods of high marginal utility—as times when interest rates fall causing bond prices to rise. This would make bonds a hedge, not a risky asset. The fact that bonds command a risk premium is therefore surprising; and often referred to as the "bond premium puzzle". Resolving it requires a model in which the pricing kernel is negatively autocorrelated (Backus and Zin (1994)).

Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2009) considered another endowment economy model with a pricing kernel derived from utility maximization that does however account for positive term premia.[15] Their story is that it is inflation that makes nominal bonds risky, and this is indeed a

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[14] Note that the models of Capmbell (1986), Donaldson, Johnson and Mehra (1990) and Den Haan (1995) are all silent on inflation. They are models that are concerned with the real part of the term structure.

[15] The model of Bansal and Shaliastovich (2009) has the additional feature of allowing the variance of shocks to change over time, which is appealing because one can differentiate between changes in the "price" and "quantity" of risk.

recurrent theme of much recent work on the fundamental macroeconomic story that underlies bond risk premia. Piazzesi and Schneider show empirically that there is a low-frequency negative covariance between consumption growth and inflation.[16] Inflation therefore erodes the value of nominal bonds in precisely those states of the world in which consumption growth is low and so marginal utility is high. The utility function that is used is that of Epstein and Zin (1989), which is an extension of the standard power utility function in equation (19) that breaks the link between the coefficient of risk aversion and the intertemporal elasticity of substitution implied by that utility function. Epstein-Zin preferences allow an individual to be both risk-averse and yet somewhat willing to smooth consumption intertemporally, which appears to better fit agents' behavior. Using these preferences magnifies the premium that investors demand for the risk of inflation eroding the value of their nominal bonds at times when marginal utility is high, and so explains the large term premia that are observed in the data.

Since it is inflation that makes nominal bonds risky, the explanation of Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2009) implies that while the nominal yield curve ought to slope up, the real yield curve should be roughly flat or even slope down. As pointed out by Piazzesi and Schneider, this matches the observed average slope of nominal and real yield curves in the U.K., but not in the U.S.[17]

Ulrich (2010) appeals to Knightian uncertainty to give a further twist on the role of inflation in pricing nominal bonds. He considers an endowment economy in which there is uncertainty about the data generating process for inflation.

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[16] More precisely, consumption growth and inflation are both specified to be the sum of their expected values plus noise. The expected values are assumed to be slowly varying. Piazzesi and Schneider (2006) use a Kalman filter to estimate the covariance between the expected values of consumption growth and inflation, and find it to be negative.

[17] In the U.S., the TIPS yield curve is on average a bit flatter than its nominal counterpart, but it typically slopes up.

Faced with this model uncertainty, Ulrich follows the standard approach from the robust control literature, which is to suppose that agents assume the worst. That is, they price bonds assuming that inflation will be generated by whichever model *minimizes* their expected utilities. Not surprisingly, the effect of this model uncertainty is to further raise the yields that investors require to induce them to hold nominal bonds.

Wachter (2006) considers another endowment economy with consumption growth and inflation as exogenous state variables, and explicit utility maximization. The utility function is however different in that it incorporates habit formation. The investor's utility function depends not on consumption as in equation (19) but rather on consumption relative to some reference level to which the agent has become accustomed. When calibrated using U.S. data, Wachter predicts that both nominal and real yield curves slope up. The intuition is that when consumption falls, investors wish to preserve their previous level of consumption and so the price of bonds goes down as marginal utility rises. This makes bonds (real or nominal) bad hedges, as they do badly when investors need them the most, and leads them to command positive risk premia in equilibrium. Wachter also finds that the model can match other term premium puzzles, notably the negative slope in the estimation of equation (8).

4.4 Structural models for the pricing kernel and factor dynamics

Subsection 4.2 used structural models for the factor dynamics and a statistical representation for the pricing kernel. Subsection 4.3 did exactly the opposite. But recently some authors have used structural models for both the factor dynamics and the pricing kernel, and this is the logical conclusion of a progression from atheoretical to structural models. For example, Bekaert, Cho and Moreno

(2010) combine a forward looking new-Keynesian model with a stochastic discount factor derived from maximizing utility in equation (19). The model is loglinear and lognormal, which makes it tractable to solve, but which however implies that the expectations hypothesis holds and that there is no term premium (apart from the Jensen's inequality effect). A general problem with a structural model for both the pricing kernel and the factor dynamics is that it is challenging to maintain computational tractability and yet obtain time-variation in term premia.[18]

Rudebusch, Sack and Swanson (2007) and Rudebusch and Swanson (2008) do however model time-varying term premia in DSGE models with production using preferences with habit formation (as considered by Wachter (2006) in the context of an endowment economy). They find that the success that Wachter obtained in using habits to explain bond risk premia in an endowment economy does not extend to a DSGE model. The term premia in a habit-based DSGE model are very small. The intuition is that whereas in an endowment economy, agents facing a negative consumption shock will wish to sell bonds to smooth their consumption, in a production economy they can and will choose to raise their labor supply instead (Swanson, 2010).[19]

Rudebusch and Swanson (2009) did a similar exercise but using Epstein-Zin preferences instead. They had much more success in matching the basic empirical properties of the term structure. The intuition is an extension of that of Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2008) to a

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[18] These models require solution methods that are based on approximations around a non-stochastic steady-state. A first-order approximation delivers a zero term premium—it is as though agents were risk-neutral. A second-order approximation delivers a constant term premium. Only with a third-order approximation, considered by Rudebusch, Sack and Swanson (2007), Rudebusch and Swanson (2008, 2009), Ravenna and Seppälä (2007) and Van Binsbergen, Fernández-Villaverde, Koijen and Rubio-Ramírez (2008) does it become possible to have time-varying term premia.

[19] Alternatively, Rudebusch and Swanson (2008) can match the term premium in the habit-based DSGE model, but at the price of making real wages far more volatile than is actually the case in the data.

production economy: technology shocks cause consumption growth and inflation to move in opposite directions, meaning that inflation will erode the value of nominal bonds in precisely the state of the world when investors' marginal utility is high. This makes nominal bonds command a positive risk premium.

4.5 Inflation hedging as the cause of term premia

The last two subsections have reviewed a range of macro-finance term structure models in which the pricing kernel comes from an explicit utility-maximization problem. These models are all quite different. Yet many of them agree on one thing—inflation uncertainty makes nominal bonds risky. Although the search for fundamental macroeconomic-based explanations for term premia remains a work in progress, this does seem to be a pattern found by many authors.

If investors demand positive term premia to hedge against inflation risk, then we would expect inflation and consumption growth to move in opposite directions (as Piazzesi and Schneider (2006) and others have found empirically). We'd also expect a positive correlation between nominal bond returns and consumption growth, or other real-side measures. Campbell, Sunderam and Viceira (2007) found that the correlation between nominal bond returns and the real economy has varied over time, but was particularly high during the period of high inflation in the 1970s and early 1980s (the "Great Inflation"). They also pointed out that the average slope of the yield curve has been unstable over time—yield curves tended to be fairly flat before the early 1970s, then became steep, and then flattened once again since the mid 1990s (see also Fama (2006)). Tellingly, these two shifts line up to some extent—the yield curve was steepest at the time when nominal bonds were especially risky assets. This pattern could indeed help to account for the bond premium puzzle, and for time-variation in term premia. According to this story, in the U.S. over most of the last few decades, investors have mainly been concerned about supply shocks that shift the Phillips curve in and out, and they have consequently demanded positive bond risk premia. But the size, and even the sign, of bond risk premia depend on the economic environment. If investors were instead, at some times, more concerned about demand shocks shifting the economy along the Phillips curve, then they would view nominal bonds as a good hedge, and bond risk premia would be negative. Perhaps this helps explain the low level of bond yields in the summer of 2010—investors may have viewed bonds as a good hedge against the possibility of deflation and sustained economic weakness.

Piazzesi and Schneider (2006) also argued that term premia were particularly large during and immediately after the Great Inflation, because the long-run correlation between inflation and consumption growth was especially negative at this time. Meanwhile, they argue that at other times, the relative importance of inflation shocks in the economy was smaller, and term premia were apparently lower. Palomino (2008) goes further back in time, and documents that the average term spread was negative in the U.S. under the Gold Standard from 1880 to 1932, which he interprets as evidence that the term premium reflects instability in long-term inflation expectations. The relatively favorable evidence on the expectations hypothesis from this period, and from other countries that arguably have more stable long-run inflation expectations (discussed in section 3 above), also supports this view.

Rudebusch, Swanson and Wu (2007) found that many affine term structure models showed a downward trend in estimated term premia over the course of the 1990s. This pattern is clearly visible in Figure 6 of this paper. A natural interpretation is that the 1990s were a time when inflation uncertainty was waning, again suggesting that inflation uncertainty is a key driver of bond risk premia.

There is yet more evidence to support this broad conclusion. A compelling example, is the market reaction to the announcement that the Bank of England was to be granted operational independence, on May 6, 1997. As documented by Gürkaynak, Levin and Swanson (2010) and Wright (2010), U.K. nominal yields fell sharply, and the nominal yield curve flattened dramatically, on the very day of this announcement. Meanwhile, real yields were little changed. It seems hard to account for this without appealing to the idea that a more stable nominal anchor lowered both inflation expectations and inflation risk premia.

On the other hand, a note of caution with respect to the view that inflation uncertainty is the cause of term premia is that this may be hard to reconcile with the patterns observed so far in the relatively new and comparatively illiquid U.S. TIPS market. Under this view, one might expect the real yield curve to be flat or to slope down, but in fact the TIPS yield curve typically slopes up. And long-term TIPS forward rates have moved almost in lockstep with their nominal counterparts (as shown in Figure 5).

4.6 Affine models with both nominal and index-linked bonds

A few recent papers have undertaken the ambitious but important task of applying the affine model framework to nominal and index-linked bonds jointly. Let $P_t^{REAL}(n)$ be the real price of an index-linked zero-coupon n-period bond at time t, and let $Q(t)$ be the price level at time t. The analog of equation (5) is then:

$$P_t^{REAL}(n) = E_t(\textstyle\prod_{i=1}^{n} M_{t+i}^{REAL}) \tag{20}$$

where $M_{t+i}^{REAL} = \frac{Q(t+i)}{Q(t+i-1)} M_{t+i}$ is the real pricing kernel. Coupled with an assumption that the inflation rate is of the form

$$\ln(Q(t+1)/Q(t)) = \mu_0 + \mu_1' X_t + \xi_{t+1}$$

where ξ_t is iid $N(0, \sigma_\xi^2)$,[20] (perhaps correlated with the factor innovations ε_{t+1}), equation (20) implies that real yields will be an affine function of the state vector X_t, similar to equation (16). Several authors have fitted such a model to nominal and TIPS yields jointly, including Buraschi and Jiltsov (2005), Kim (2004), D'Amico, Kim and Wei (2010) and Christensen, Lopez and Rudebusch (2010). In this way, in addition to having a decomposition of the nominal yield into nominal expected short rates and a nominal term premium, one can also decompose the *real* yield into *real* expected short rates and a *real* term premium. And then, as a matter of arithmetic, the difference between these two is the decomposition of breakeven inflation[21] into inflation expectations and an inflation risk premium. In other words, the nominal yield is decomposed into four components: the expected real rate, expected inflation, the real risk premium and the inflation risk premium.

In the U.S., the TIPS market is tiny relative to the vast nominal Treasury market. At present, daily trading volumes in TIPS run at 1-2 percent of their nominal counterparts.[22] Liquidity in the TIPS market was very poor in the years immediately following the launch of the TIPS program in 1997, and indeed at times there was talk of the index-linked bond issuance being discontinued in the United States. TIPS liquidity improved over the subsequent years, but then worsened sharply during the financial crisis (see, for example, Campbell, Shiller and Viceira (2009)). Investors surely demand a higher yield on TIPS to compensate them for this comparative lack of liquidity, and this liquidity premium must vary over time. In particular, it is impossible to rationalize the high level of TIPS yields during the financial crisis without appeal to a sizeable

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[20] This represents a decomposition of inflation into *expected inflation*, $\mu_0 + \mu_1' X_t$, and *unexpected inflation*, ξ_t.

[21] Recall that breakeven inflation is defined as the spread between comparable maturity nominal and real bond yields.

[22] Source: Federal Reserve Bank of New York Survey of Primary Dealers.

liquidity premium.[23] D'Amico, Kim and Wei (2010) argue more broadly that a time-varying liquidity premium needs to be taken out of TIPS yields before using them to fit an affine term structure model. Such efforts will be especially useful when studying the behavior of real and inflation related components of the term structure during times of crises.

5 Learning about Structural Change

The models discussed in section 4 assume parameter constancy. And yet, these models are estimated over a period of time in which many macroeconomists believe that there were important changes in the economy, notably changes in the Fed's implicit inflation target, that agents learned about slowly. Stock and Watson (2007) and Cogley, Primiceri and Sargent (2009) argue that the permanent component of U.S. inflation—or the Fed's implicit inflation target—varied considerably over the last 40 years.

The idea that investors learn slowly about structural change has been incorporated into models of the term structure. Indeed, some authors argue that this may account in large part for the apparent failure of the EH. For example, Kozicki and Tinsley (2005) consider a model in which long-term interest rates are indeed given by agents' beliefs about expected average future short rates, but in which these beliefs are conditioned on their perceptions of the central bank's long-run-inflation target, not the true inflation target. These perceptions of the long-run inflation target are in turn formed by backward-looking adaptive expectations. This means that agents make systematic forecasting errors for inflation, and hence interest rates.[24]

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[23] Certain TIPS real yields were noticeably above comparable maturity nominal yields at times during the fall of 2008. While low inflation expectations (and fear of deflation) no doubt contributed to this, the indexation adjustment to TIPS principal cannot be negative. For this reason, when TIPS yields are above their nominal counterparts, this can only represent a liquidity premium.

[24] Even if agents did not know the true inflation target, they could still form expectations

Kozicki and Tinsley argue that this model can explain the key term structure anomalies. For example, if there is a downward shift in the true inflation target, current inflation will be below investors' long-run inflation expectations, and the yield curve will be steep. As investors learn slowly about the change in the inflation target, the yield on long-term bonds will decline, explaining the tendency of a *steep* yield curve to be followed by *falling* long-term rates (Table 2). The fact that the sample periods and countries for which the EH finds most support are those for which long-run inflation expectations seem likely to be stable, lends support to this explanation. Other papers also attribute term structure anomalies at least in part to shifting perceptions of the central bank's implicit inflation target, including Kozicki and Tinsley (2001), Gürkaynak, Sack and Swanson (2005), Rudebusch and Wu (2007), Erceg and Levin (2003), Dewachter and Lyrio (2006) and De Graeve, Emiris and Wouters (2009).[25]

Learning about structural change can also be incorporated into affine term structure models. One simple approach is to take any standard term structure model, but to re-estimate it in each period using a rolling window of data (such as the last ten years, or using some pattern of declining weights). The idea is to mimic the behavior of an economic agent who learns about parameter values from the recent past ("constant gain learning"). Laubach, Tetlow and Williams (2007) and Orphanides and Wei (2010) estimated affine term structure

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for it conditional on the available information. Were they to do so, by construction, there could be no predictable forecast errors. The systematic forecast errors in the model of Kozicki and Tinsley (2005) require agents to both be unaware of the true inflation target and to form beliefs for this inflation target that do not make full use of the information that they have.

[25] De Graeve, Emiris and Wouters (2009) consider a DSGE model of the sort proposed by Smets and Wouters (2007). The model imposes the expectations hypothesis, except allowing for constant term premia for bonds of each maturity which are free parameters and not explained by the model. It also allows for a time-varying inflation target. The model turns out to give out-of-sample forecasts of yields that are competitive with a number of standard benchmarks. Note that this model does not provide a structural explanation for the bond premium puzzle (unlike some of the models discussed in section 4), because it treats the average term premia at each maturity as free parameters, rather than being endogenously determined by the model.

models using rolling windows of data. A variant on this in which parameters are estimated with geometrically declining weights on older data was considered by Piazzesi and Schneider (2006). These methods have the advantage of allowing for learning about many different structural breaks, not just changes in the inflation objective.[26] But using rolling windows of data to estimate models is not ideal either, because of course the results can be sensitive to arbitrary choices of the window size.

Another simple approach for estimating term premia that is robust to learning and structural breaks is to use the difference between long-term interest rates and survey measures of interest rate expectations as term premium estimates. This seems very appealing—if surveys are at least approximating investors' expectations, then this is essentially the ideal way of parsing long-term interest rates into term premium and expected future short rate components. Piazzesi and Schneider (2008) and Wright (2010) estimate term premia in this way. Surveys probably do a good job of capturing secular shifts in inflation expectations, though it's hard to argue that they are perfect measures of expectations. For one thing, they sometimes seem to be implausibly inertial.

6 Preferred Habitats

The affine term structure models of section 4 all seek to match the cross-sectional and time-series patterns of interest rates within a single coherent asset pricing framework. Meanwhile, the expectations hypothesis and these affine term structure models both agree on one point: changes in the supply of bonds should affect yields only to the extent that either expectations of future short-term in-

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[26] Aside from changes in the implicit inflation target, another kind of structural change that is often considered is the possibility that the sensitivity of the Fed to deviations of inflation from target in the Taylor rule has changed over time. Ang, Boivin and Dong (2008), Bikbov and Chernov (2005) and Smith and Taylor (2009) consider affine term structure models with this form of instability and find supportive evidence for this view.

terest rates, or the factors in the model, are changed. Eggertson and Woodford (2003) argued that central bank asset purchases could affect yields, but only if they changed market expectations of the future path of monetary policy.[27]

An alternative paradigm is the preferred habitat theory of Modigliani and Sutch (1966, 1967) in which markets are segmented, investors demand bonds of a specific maturity, and the interest rate is determined by the supply and demand of bonds of that particular maturity. Until recently, this view did not find much favor in academic research. Part of this was because theoretical models made it hard to justify the reluctance of arbitrageurs to effectively integrate markets. It also owed in part to the fact that Operation Twist in the 1960s—whereby the U.S. Treasury shortened the maturity structure of outstanding debt with the aim of raising short term interest rates while lowering long-term rates— was generally seen as ineffective (Modigliani and Sutch (1966, 1967)).

But Operation Twist was a small program. And empirical evidence has been kinder to the preferred habitats view of late. In 2000 and 2001, the Treasury conducted buy-backs of longer-term Treasury securities. Bernanke, Reinhart and Sack (2004) argued that these operations had a sizeable effect on the term structure. Another example is that in the United Kingdom, pension funds face strict rules requiring them to hold long-term government securities. Long-term bond yields have been especially low in the United Kingdom since these rules came into force, with the real yield on fifty-year indexed government bonds in the U.K. falling below half a percentage point at one time. Special demand from pension funds is a natural explanation for these exceptionally low yields (Bank of England (1999) and Greenwood and Vayanos (2010)).[28]

Some papers have looked at the long-term empirical relationship between

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[27] Dai and Philippon (2005) consider an affine model with a measure of fiscal policy as an element of the state vector, which provides a clear channel for bond supply to affect yields.

[28] Some authors have argued that Greenspan's "conundrum" was largely the result of demand for U.S. Treasury securities from foreign central banks (e.g. Craine and Martin (2009)). Others, such as Rudebusch, Swanson and Wu (2006) disagree with this finding.

yield spreads and measures of the effective supply of debt. Krishnamurthy and Vissing-Jorgensen (2008) found that the higher is the supply of government debt, the larger is the spread between the very highest quality corporate bonds and comparable maturity government bonds, consistent with the idea that some investors have a special preference for sovereign bonds rather than close substitutes. Krishnamurthy and Vissing-Jorgensen argue that government bonds are virtually unique among assets in having a degree of liquidity and safety that makes them in effect close substitutes for money. Thus the prices of government bonds should be affected by their supply and by the demand for the special money-like characteristics of sovereign debt, rather than simply being determined in a multi-factor asset pricing model. In the same spirit, Kuttner (2006) and Greenwood and Vayanos (2008) both ran augmented regressions of the form of equation (9) with measures of the maturity of outstanding Treasury debt. They found that a larger supply of outstanding long-term debt was associated with a higher bond risk premium.

There are also a number of "event studies" which show that announcements of changes in the supply of a particular class of securities are associated with changes in the prices of those securities. For example, as documented by Bernanke, Reinhart and Sack (2004), on the day in the fall of 2000 when the Treasury announced that it was ceasing new issuance of thirty-year bonds, the yields on these bonds fell sharply. Or, on the day in March 2009 when the Federal Reserve announced that it was buying $300 billion in Treasury securities, long-term Treasury yields dropped by about 50 basis points (Gagnon, Raskin, Remache and Sack (2010)).[29]

Figure 7 shows the yields of outstanding Treasury coupon securities on four

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[29] In the subsequent weeks, long-term Treasury yields rebounded. It is impossible to know if this was because the effect of the Federal Reserve announcement "wore off", or if Treasury yields would have climbed anyway on better-than-expected incoming economic data. Doh (2010) notes however that a good bit of the rebound in yields occurred right around macroeconomic news announcements, tending to support the latter interpretation.

recent days: January 1, 2008, two dates during the depths of the recent financial crisis, and the last day in 2009. Note that these are the actual yields-to-maturity on individual securities, as opposed to the smoothed yield curves that have been used elsewhere throughout this paper. In normal times, yields are a smooth function of time-to-maturity. In Figure 7, we can see that this was the case at the beginning of 2008, and again at the end of 2009. But, in contrast, in November and December 2008, comparable maturity bonds were trading at quite different yields.[30] Thirty-year bonds that had been issued in the late 1980s and that had about 7 years left to maturity had substantially higher yields than ten-year notes with the same time to maturity.[31] Ordinarily, arbitrageurs should make such discrepancies vanish in an instant. Summers (1985) commented that financial economics amounted to checking that two quart bottles of ketchup sell for twice as much as one quart bottles of ketchup. In the fall of 2008, they did not. This is very vivid evidence of market segmentation, and is a challenge to bond pricing models that rely heavily on the assumption that investors do not leave arbitrage opportunities on the table.[32]

All these empirical facts motivate a good theoretical explanation. Work by Vayanos and Vila (2009) and Greenwood and Vayanos (2008) has begun to fill this void. These models have three groups of agents: the government, investors, and arbitrageurs. The government issues bonds and the investors have demand

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[30] Much smaller divergences in yields of securities with the same maturity date were noticed around the time of the collapse of Long Term Capital Management, and in the wake of September 11, 2001.

[31] It should be noted that even in a normally functioning market, bonds with different coupons maturing on the same date need not have exactly the same yields. Bonds with high coupon rates are effectively front-loading payments to the investors. When the yield curve slopes up, bonds with relatively high coupons should then have lower yields. But this does not explain the gap between old thirty-year bond yields and the yields on more recently issued ten-year notes, in Figure 7. Indeed, it goes the wrong way. During the crisis, the old thirty-year bond yields were *higher* than the ten-year note yields. Since the yield curve was upward sloping and thirty-year bonds have high coupons, in a normally functioning market, the old thirty-year bonds would have slightly *lower* yields.

[32] A "movie" showing the yield curve day-by-day in 2008 and 2009 is available at http://www.econ.jhu.edu/People/Wright/loop_repealed.html. Figure 7 is effectively giving four frames from this movie.

for these bonds. The "net supply" of bonds of maturity n (meaning the supply of bonds issued by the government less the demand for those bonds from investors) is:

$$s_t(n) = \alpha(n)[\beta_t(n) - y_t(n)] \tag{21}$$

where $\alpha(n) > 0$ and $\beta_t(n)$ is an exogenous stochastic process. The process $\beta_t(n)$ represents the yield at which the net supply of bonds of maturity n will be zero (i.e. all bonds issued by the government are bought by investors). This reflects the tastes of the government and investors for bonds of this maturity. Meanwhile $\alpha(n)$ measures the sensitivity of the government supply and/or investor demand for these bonds to changes in their yield. If the yield, $y_t(n)$, goes up, then equation (21) means that the net supply of bonds of maturity n goes down, as the government will issue less of this bond and/or the investors will demand more of it. Meanwhile, the short-term interest rate, $y_t(1)$, follows an unrelated exogenous stochastic process. The arbitrageurs maximize a utility function that depends on the mean and variance of their wealth; the more risk-averse the arbitrageurs, the more disutility they get from variance. For the market to clear, it must be the case that the demand of the arbitrageurs for bonds of maturity n is equal to $s_t(n)$. If the arbitrageurs were risk-neutral, then they would entirely undo the effects of market segmentation in equation (21), and the yield would just be the average future expected short-term interest rate. In the opposite limiting case, if the arbitrageurs were infinitely risk-averse, then they would not participate in the market and it would be that case that $y_t(n) = \beta_t(n)$ in equilibrium. Bond markets would be completely segmented, and changes in yields at one maturity would be irrelevant for yields at all other maturities. For intermediate cases, yields are determined both by expectations of future short-term interest rates, but also by the demand of investors for bonds of particular maturities. Intuitively, the arbitrageurs are balancing the potential profits from buying a cheap bond against the risk that a shock to $\beta_t(n)$ will make this bond even cheaper, causing them to lose money.

Although these papers do not explicitly model it, one might suppose that the risk aversion of arbitrageurs increases at times of financial crisis, amplifying the importance of market segmentation at these times. That would mean that shifts in the net supply of bonds would have larger effects on yields at times of market stress than at times of more normal market functioning.

A belief in the preferred habitats view evidently motivated the large-scale asset purchases of mortgage-backed-securities, Treasury securities and other debt by the Federal Reserve and other central banks during the recent financial crisis. Federal Reserve vice-Chairman Kohn pointed to the preferred habitat framework as guiding their decision (Kohn (2009)). The Federal Reserve set its policy interest rate to around zero, and yet was concerned that more actions to support aggregate demand were needed to avoid the economy being stuck in a liquidity trap. To this end, the Fed also expressed the intention of leaving the short-term policy rate at this level for a long period. Under the EH, only actions that change the current or expected future stance of monetary policy should alter longer-term interest rates.[33] But under the preferred habitat view, changing the net supply of fixed income assets should have a direct effect on their market price.

Turning to empirical evidence, Gagnon, Raskin, Remache and Sack (2010) argued that large-scale asset purchases by the Federal Reserve did indeed substantially lower long-term benchmark interest rates, including yields on both Treasuries and mortgage-backed-securities. D'Amico and King (2010) reached similar conclusions, comparing the prices of securities that the Federal Reserve purchased with those that it did not buy. Hamilton and Wu (2010), estimating

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[33] And in affine term structure models, only actions that affect the factors should matter for the term structure.

a model based on the framework of Vayanos and Vila (2009) over pre-crisis period data, also concluded that the Federal Reserve has the potential to rotate the yield curve through its asset purchases. Kohn (2009) judged that the Federal Reserve's large-scale asset purchases had resulted in "cumulative restraint on the average level of longer-term interest rates, perhaps by as much as 100 basis points." These purchases stopped in early 2010.[34] Although the announcements of large-scale asset purchases by the Federal Reserve in 2008 and 2009 were accompanied by sharp drops in fixed income yields, the news that these purchases were being ended did not elicit comparable rises in rates. This may be because the termination of the programs was expected by investors and/or because shifts in the net supply of bonds have much larger effects during crises than at times of normal market functioning.

All in all, while the standard affine term structure model seems to be the most appealing framework for understanding yield curve movements in normal times, the preferred habitat approach seems also to have value, especially at times of unusual financial market turmoil.

7 Conclusions

In post-war U.S. data, the upward slope of the yield curve is hard to miss—and to explain. This bond premium puzzle seems at least as important as the equity premium puzzle. As Rudebusch and Swanson (2008) observe, the value of long-term bonds in the U.S. far exceeds that of equities. Yet the attention given to the equity premium puzzle was far greater, until recently. Also, the available evidence points to predictable time-variation in these bond risk premia.

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[34] At the time of writing, the Federal Reserve has resumed asset purchases, but only to the extent of reinvesting the proceeds of maturing securities. FOMC members were discussing the possibility of expanding the size of its balance sheet further, should the economic recovery falter. But they were also considering asset *sales*, in the event of the recovery proving to be robust.

A great deal of work has been undertaken in the last two decades that accounts for these patterns in the term structure of interest rates. Affine term structure models have been shown to be a powerful tool for explaining term structure anomalies within an internally consistent asset-pricing framework, and can moreover include structural economic foundations. Although the quest for the fundamental macroeconomic explanations of bond risk premia is still ongoing, a common theme of much of the work in the macro-finance literature is that it is inflation uncertainty that makes nominal bonds risky. This means that measures to stabilize long-run inflation expectations should make risk premia on long-term bonds both lower and more stable. It would thus make the Treasury's borrowing costs on longer-term debt both lower and more predictable.

It should be emphasized that the instability in investors' inflation expectations that appears to be a large part of the story underlying bond risk premia does not necessarily have to result from central banks constantly altering their fundamental preferences regarding inflation. It could also come from a lack of central bank credibility that might for a time drive a wedge between actual and perceived inflation targets. Or it could come about as a result of shocks on the real side of the economy. For example, the recession of 1990-1991 created slack in the economy that put downward pressure on inflation. The Fed had not been willing to deliberately create a recession in order to bring this about, but was nonetheless happy to accept this "opportunistic disinflation" and made no attempt to reverse it subsequently. Likewise investors may think that the aftermath of the recession that began in December 2007 could result in a higher level of inflation for a very extended period that the Federal Reserve might ultimately regard as tolerable, even if not ideal.

Affine term structure models exploit the predictability of interest rates while respecting the principle that investors leave no arbitrage opportunities on the table. Both expected short rates and term premia can be tied to (observable or latent) economic fundamentals within this framework and the yields can be decomposed into expected rates and term premia to make policy relevant inferences. It generally seems reasonable and appropriate to impose an absence of arbitrage; investors are normally very quick to eat a free lunch. But the potential for market segmentation has been highlighted by the recent financial crisis, and preferred habitat models are enjoying a renaissance. At the depths of the crisis, even the prices of the simplest fixed income securities were apparently not mutually consistent. This has a number of important potential implications. One is that it creates a rationale for large-scale asset purchases by the central bank. Another is that it calls sharply into question the value of exercises of finding the "market price" of especially opaque and illiquid securities.

The behavior of long-term interest rates was part of the backdrop to the recent financial crisis (Greenspan's conundrum) and was integral to the response to the crisis as the Federal Reserve and other central banks sought to drive down longer-term rates after they had pushed overnight interest rates to the zero bound. As part of the exit strategy from this unusual period, the Federal Reserve and other central banks will continue to want to influence the term structure of rates and to measure macroeconomic expectations from the yield curve. To date, there are few signs of the crisis leading long-run inflation expectations to become unanchored. But the evidence from the macro-finance term structure literature suggests that if that were to happen in the future, then it would lead to a large rebound in term premia

Table 1: Slope Coefficient From Estimation of Equation (7)

	$m = 3$	$m = 6$	$m = 12$
n=24	0.51***	0.26***	0.25**
	(0.17)	(0.22)	(0.30)
n=48	0.74	0.61*	0.65
	(0.16)	(0.22)	(0.32)
n=72	0.81	0.72	0.82
	(0.15)	(0.19)	(0.26)
n=96	0.74	0.63*	0.74
	(0.16)	(0.20)	(0.27)
n=120	0.63**	0.51**	0.62
	(0.17)	(0.21)	(0.28)

Notes: Estimates of the slope coefficient in equation (7) for selected choices of m and n, in months. Newey-West standard errors with a lag truncation parameter of m are included in parentheses. The data are monthly, from August 1971 to December 2009. Cases in which the slope coefficient is significantly different from one at the 10, 5, and 1 percent levels are denoted with one, two, and three asterisks, respectively. Bond yields are from the dataset of Gürkaynak, Sack and Wright (2007).

Table 2: Slope Coefficient from Estimation of Equation (8)

	$m = 3$	$m = 6$	$m = 12$
n=24	-1.56***	-1.15***	-0.50**
	(0.71)	(0.62)	(0.59)
n=48	-1.90***	-1.55***	-1.03***
	(0.95)	(0.72)	(0.74)
n=72	-2.31***	-1.91***	-1.47***
	(1.06)	(0.79)	(0.84)
n=96	-2.76***	-2.24***	-1.86***
	(1.17)	(0.88)	(0.93)
n=120	-3.21***	-2.59***	-2.21***
	(1.28)	(0.98)	(1.03)

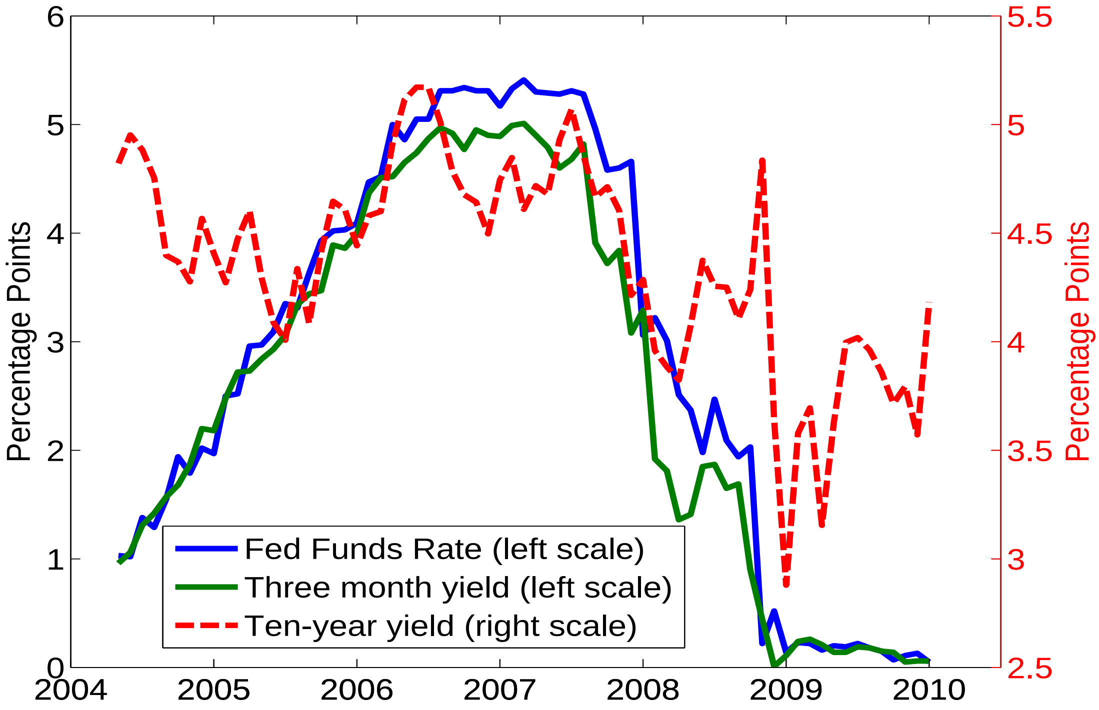
Notes: Estimates of the slope coefficient in equation (8) for selected choices of m and n, in months. Newey-West standard errors with a lag truncation parameter of m are included in parentheses. The data are monthly, from August 1971 to December 2009. Cases in which the slope coefficient is significantly different from one at the 10, 5, and 1 percent levels are denoted with one, two, and three asterisks, respectively. Bond yields are from the dataset of Gürkaynak, Sack and Wright (2007).

Table 3: Slope Coefficient From Estimation of Equation (9)

	$n = 24$	$n = 48$	$n = 72$	$n = 96$	$n = 120$
	Sample Period: August 1971-December 2009				
β_0	-0.83	-1.76	-2.88	-4.34	-6.00
	(0.96)	(2.40)	(3.53)	(4.52)	(5.47)
β_1	-0.54**	-1.68**	-2.69***	-3.61***	-4.52***
	(0.27)	(0.67)	(0.98)	(1.24)	(1.49)
β_2	0.86	2.50	3.60	4.49	5.43
	(0.72)	(1.74)	(2.51)	(3.16)	(3.75)
β_3	-0.17	-0.54	-0.52	-0.35	-0.23
	(0.57)	(1.36)	(1.98)	(2.49)	(2.97)
Wald	7.17*	8.71**	10.65**	12.46***	13.78***
R^2	0.12	0.14	0.16	0.18	0.19
	Sample Period: August 1971-December 2006				
β_0	-1.50	-3.59	-5.43	-7.31	-9.27
	(0.97)	(2.38)	(3.56)	(4.66)	(5.74)
β_1	-0.79***	-2.36***	-3.64***	-4.76***	-5.80***
	(0.25)	(0.58)	(0.86)	(1.12)	(1.39)
β_2	1.61**	4.48***	6.40***	7.84***	9.18**
	(0.71)	(1.59)	(2.28)	(2.93)	(3.60)
β_3	-0.60	-1.68	-2.13	-2.28	-2.39
	(0.58)	(1.32)	(1.88)	(2.41)	(2.93)
Wald	15.15***	19.42***	20.30***	20.32***	20.10***
R^2	0.20	0.23	0.25	0.25	0.26

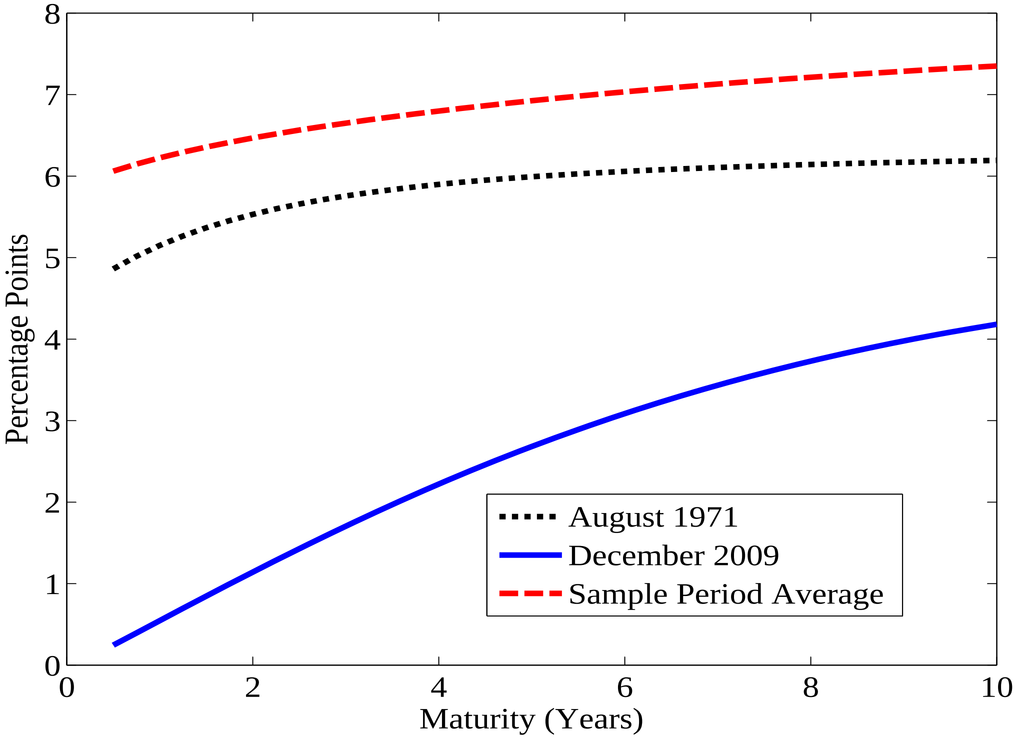
Notes: Estimates of the slope coefficient in equation (9) for selected choices of n, in months. The holding period is m =12 months in all cases. Newey-West standard errors with a lag truncation parameter of m are included in parentheses. The table also shows Wald statistics testing the hypothesis that the slope coefficients are jointly equal to zero, and the regression R^2 values. The data are monthly, from August 1971 to December 2009. Cases in which the slope coefficient/Wald statistic is significantly different from one at the 10, 5, and 1 percent levels are denoted with one, two, and three asterisks, respectively. Bond yields are from the dataset of Gürkaynak, Sack and Wright (2007).

Figure 1: Selected Interest Rates in Recent Years



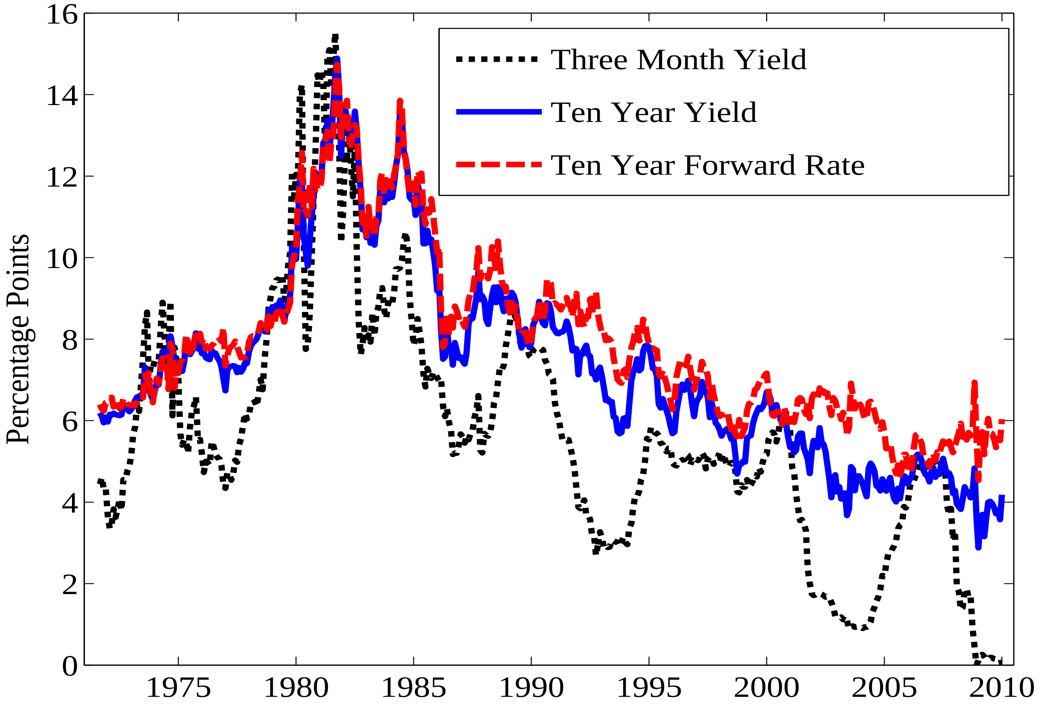
Notes: End-of-month data. The federal funds rate and three-month yield are taken from the Fed's H-15 release. The ten-year yield is a zero-coupon yield from the smoothed yield curve described in Gürkaynak, Sack and Wright (2007), the data for which are available on the Fed's website.

Figure 2: Zero-coupon nominal yield curves



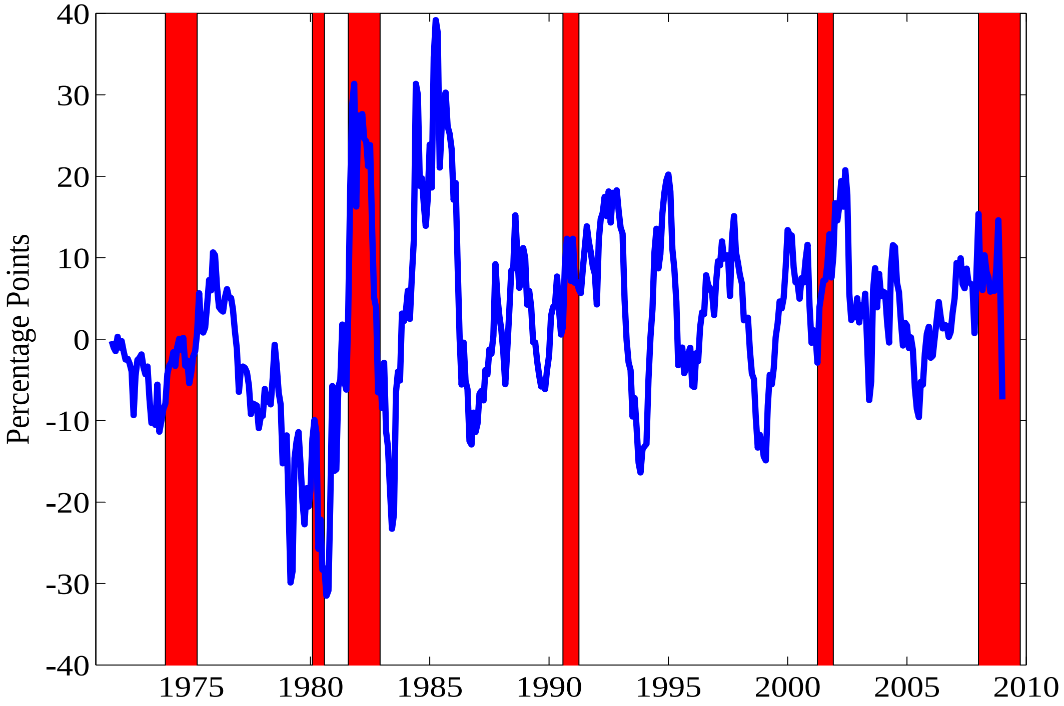
Notes: The figure shows the zero-coupon yield curves from the smoothed yield curve of Gürkaynak, Sack and Wright (2007) at the end of August 1971, the end of December 2009, and averaging across all months in between.

Figure 3: Bond Yields and Forward Rates Since 1984



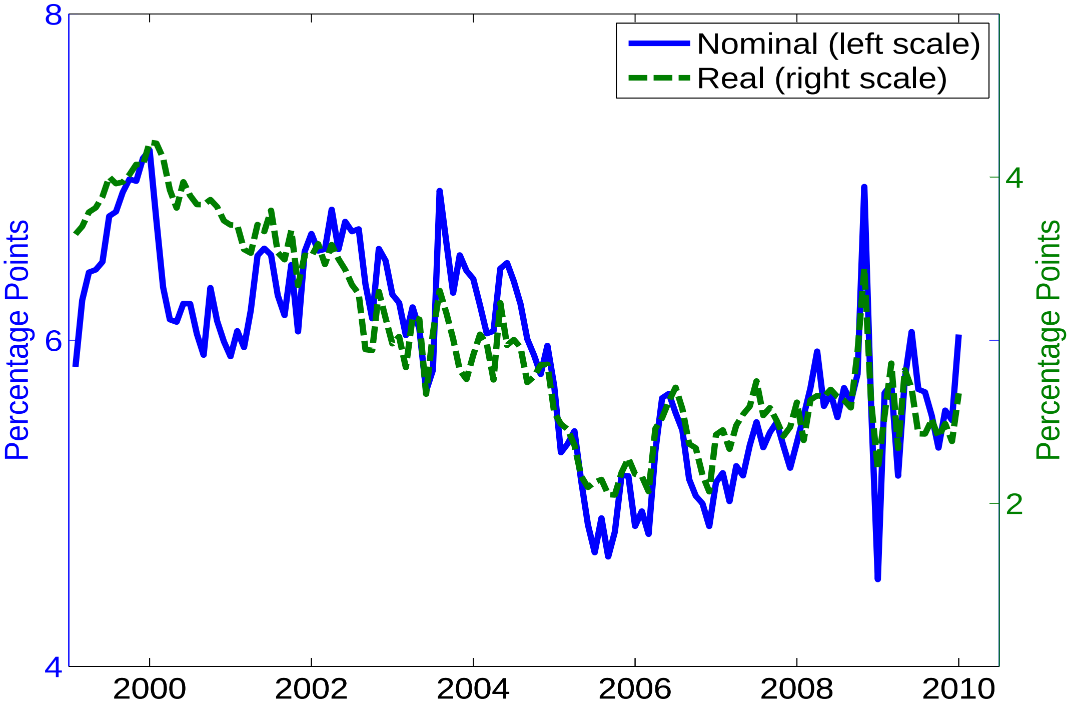
Notes: End-of-month data. The three-month yield is taken from the Fed's H-15 release. The ten-year yield is a zero-coupon yield from the smoothed yield curve of Gürkaynak, Sack and Wright (2007), and the ten-year forward rate is an instantaneous forward rate from the same source.

Figure 4: One-year Excess holding period returns of the ten-year over the one-year bonds



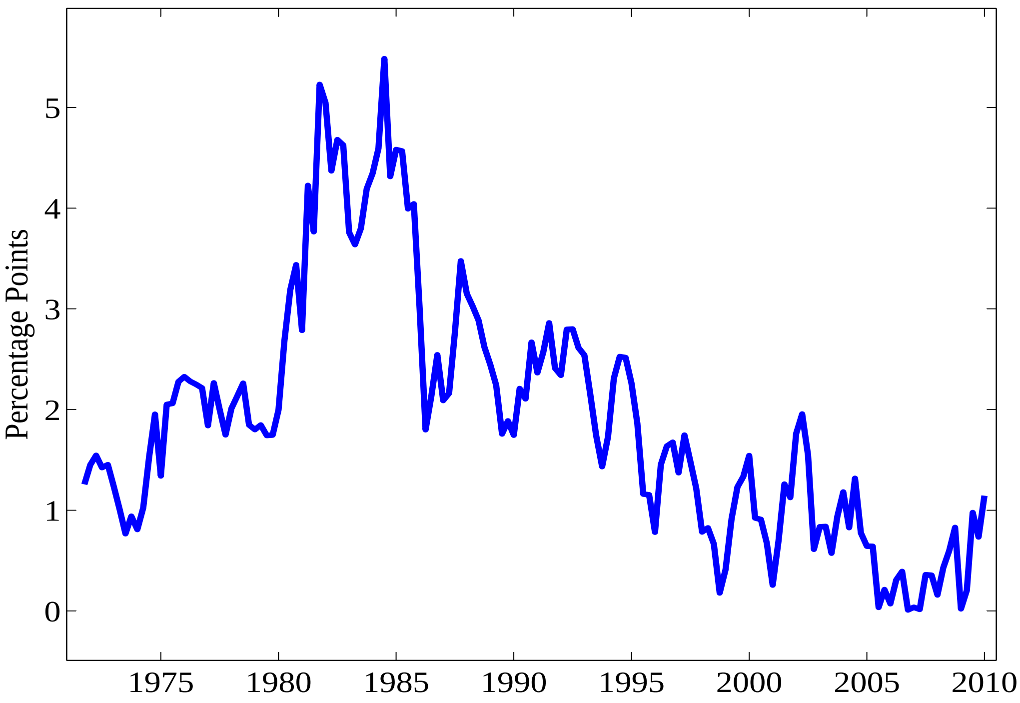
Notes: The figure plots the excess returns on holding a ten-year bond over the return on holding a one-year bond, for a holding period of one year. Returns are plotted against the date at the *end* of the holding period. NBER recession dates are shaded. Bond returns are computed using the zero-coupon yields from the smoothed yield curve of Gürkaynak, Sack and Wright (2007).

Figure 5: Slope of Nominal and TIPS Yield curves



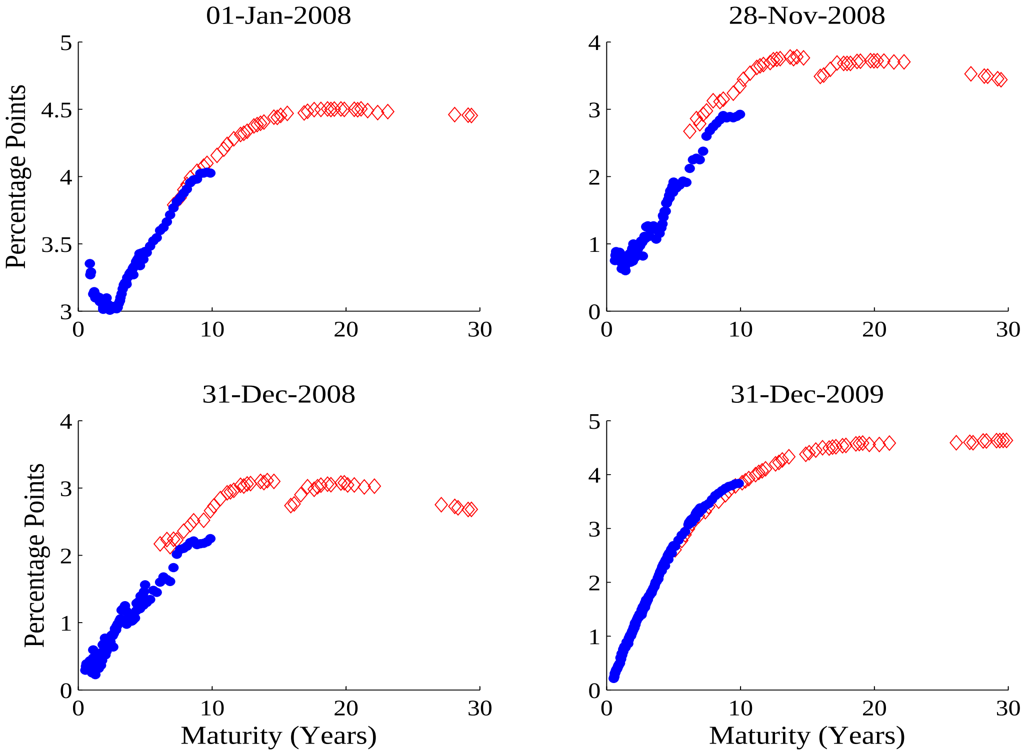
Notes: This graph plots the spread between ten- and five-year TIPS yields (January 1999-December 2009) and also the spread between ten- and five-year nominal Treasury yields. All yields are zero-coupon yields from the smoothed yield curves described in Gürkaynak, Sack and Wright (2007, 2010), the data for which are available on the Fed's website.

Figure 6: Ten-Year Term Premium Estimate



Notes: Estimate of the ten-year term premium from the model of Christensen, Diebold and Rudebusch (2007). The data used in the model were end-of-quarter 3 month, 6 month, and 1, 2, through 10 year yields. The 3 month and 6 month yields are from the Fed's H-15 release. The remaining yields are zero-coupon yield from the smoothed yield curve of Gürkaynak, Sack and Wright (2007). The model was estimated by the Kalman filter, with the measurement equation being given by (24) and the transition equation being a VAR(1) for the state vector $X_t = (X_{1t}, X_{2t}, X_{3t})'$. The sample period is 1971Q3-2009Q4.

Figure 7: Yields on Treasury coupon securities on selected dates



Notes: Yields-to-maturity on outstanding Treasury coupon securities, plotted against time-to-maturity on four recent dates. Securities originally issued as thirty-year bonds are represented as diamonds; all other securities are plotted as solid circles.

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