The TIPS Yield Curve and Inflation Compensation[*]

Refet S. Gürkaynak

Brian Sack

and

Jonathan H. Wright[**]

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Abstract

For over ten years, the U.S. Treasury has issued index-linked debt. Federal Reserve Board staff have fitted a yield curve to these indexed securities at the daily frequency from the start of 1999 to the present. This paper describes the methodology that is used and makes the estimates public at http://www/federalreserve.gov/ econresdata/researchdata.htm. These are being updated periodically. Comparison with the corresponding nominal yield curve allows measures of inflation compensation (or breakeven inflation rates) to be computed. We discuss the interpretation of inflation compensation and provide evidence that it is not a pure measure of inflation expectations, being distorted by inflation risk premium and liquidity premium components. We estimate the liquidity premium in TIPS and find that it was especially large both in the early years of the sample and during the 2007-2008 financial market turmoil. We also propose a measure of inflation expectations, that attempts the challenging task of purging inflation compensation of its risk premium and liquidity premium components.

[**] Gürkaynak: Department of Economics, Bilkent University, 06800 Ankara, Turkey; refet@bilkent.edu.tr
  Sack: Macroeconomic Advisers, LLC, Washington DC 20006; sack@macroadvisers.com
  Wright: Department of Economics, Johns Hopkins University, Baltimore MD 21202; wrightj@jhu.edu

1. Introduction

For over ten years, the U.S. Treasury has issued Treasury inflation-protected securities (TIPS)—debt securities for which the coupon and principal payments are indexed to the Consumer Price Index (CPI)—in addition to conventional nominal bonds. However, for both nominal securities and TIPS, the Treasury issues only securities with particular maturities and coupon rates. Thus, it is not possible to directly observe the nominal or real discount factors from these issues.

In a previous paper (Gürkaynak, Sack, and Wright, 2007), we estimated a smoothed nominal Treasury yield curve from the outstanding off-the-run nominal Treasury notes and bonds. The results allowed us to compute the nominal discount factor, as well as to compute nominal Treasury yields and forward rates at any horizon. The first part of this paper is the sequel that fits a similar yield curve to outstanding TIPS. The results allow us to recover the real discount function as well as real Treasury yields and forward rates at a daily frequency back to 1999. These data are available online and are updated periodically.

A comparison of the nominal and TIPS smoothed yield curves allows us to compute measures of inflation compensation—the rate of inflation that would give an investor the same return at maturity on a nominal security and an indexed security. These measures are also known as breakeven inflation rates among financial market participants. These yield curves (nominal, TIPS, and inflation compensation) can be expressed in terms of zero-coupon yields, par yields, instantaneous forward rates, or n-by-m forward rates (that is, the m-year rate beginning n years ahead) for any n and m.

After describing how we fit the real yield curve, the paper provides a discussion on how to interpret the resulting measures. We pay particular attention to the measures of inflation compensation in this regard. These measures are often thought of as being driven by investors' expectations of inflation. However, we argue that high-frequency movements in inflation compensation are considerably too volatile to represent revisions to rational expectations of inflation alone: Inflation compensation provides information about agents' inflation expectations but its interpretation is complicated by inflation risk premia and the differential liquidity premia between TIPS and nominal securities. We provide support of this view by analyzing the time series properties of inflation compensation and actual inflation and by comparing inflation compensation to survey respondents' inflation forecasts. Among other results, we show that inflation compensation is related to the dispersion of survey forecasts, more than to the level of those forecasts, consistent with a view that a risk premium reflecting inflation uncertainty is an important element of inflation compensation.

Section 2 describes the mechanics of TIPS and briefly discusses the operation of the TIPS market. Section 3 describes the yield curve fitting exercise. Section 4 shows the results of our estimation, including an assessment of the fit of the TIPS curve. Section 5 provides empirical evidence indicating that inflation compensation is not a pure measure of inflation expectations. Section 6 delves deeper into the interpretation of inflation compensation by providing a decomposition into its components: the liquidity premium, the inflation risk premium, and inflation expectations. Section 7 offers some concluding thoughts. The yield curve data described here are updated daily and posted on the Federal Reserve Board's website.

2. The TIPS market

A nominal Treasury security pays the holder a coupon twice a year and the principal value at maturity. The coupon and principal value are fixed in nominal terms, and their real value will be eroded over time by inflation. For TIPS, on the other hand, the principal payment is multiplied by the ratio of the reference CPI on the date of maturity to the reference CPI on the date of issue.[1] If the maturity or issue date falls on day d_t of a month with d_n days, then the reference CPI is

$$CPI(-2)\frac{d_t-1}{d_n}+CPI(-3)\frac{d_n-d_t+1}{d_n}$$

where $CPI(-2)$ and $CPI(-3)$ denote the non–seasonally adjusted U.S. City Average All Items Consumer Price Index for the second and third months prior to the month in which the maturity or issue date falls, respectively. The reason for the indexation lag is that the Bureau of Labor statistics publishes these data with a lag, with the index for a given month released in the middle of the subsequent month.[2] Coupons are indexed in precisely the same way.[3] In effect, this gives the TIPS an indexation lag of about 2½ months.

The first TIPS were issued in 1997. Treasury initially sold five-, ten-, and thirty-year TIPS. The five-year TIPS was dropped in September 1998 and the thirty-year TIPS was dropped in October 2001. Subsequently, the five-year TIPS was reintroduced and a

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[1] Unless this ratio is less than one, in which case no adjustment is made. This consideration became important for newly issued bonds during the deflation scare in 2003.
[2] Barr and Campbell (1997) discuss the indexation lag for UK index-linked bonds, which was eight months for bonds issued at the time of writing of that paper.
[3] Except that the constraint that the adjustment factor cannot be less than one does not apply to the indexation of coupons. Thus a period of deflation could lower the coupon payments, but the cumulative adjustment to the principal can never be negative.

twenty-year TIPS was added in May 2004. As of the time of writing, there are 27 outstanding TIPS with maturity dates ranging from 2009 to 2032.

Liquidity in TIPS was initially poor, and investor participation in the market was limited, either due to lack of familiarity with the asset class or in some cases institutional rules preventing these securities from being held. Another important factor shaping the market was that, for a time, the long-term future of TIPS was unclear. For example, in May 2001, the Treasury Advisory Committee of the Bond Market Association recommended that the TIPS program be discontinued. However, the Treasury subsequently reaffirmed its commitment to the program, and liquidity improved substantially. TIPS now represent about 10 percent of the outstanding supply of Treasury coupon securities.[4] It is now the largest sovereign index-linked market in the world, measured in terms of the par value of outstanding issuance. More detail on the history and liquidity of the TIPS market is provided by Sack and Elsasser (2004).

3. Yield curve fitting

This section begins by reviewing the fundamental concepts of the yield curve, including the necessary "bond math" for determining both nominal and TIPS yields. It then describes the specific estimation method employed in this paper.

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[4] According to an informal survey of dealers conducted by the Federal Reserve Bank of New York in 2007, typical TIPS bid-ask spreads at maturities of five years or less were 1/2 to 1 tick (a tick is roughly 1/32nd of a percentage point of the price of the security). At maturities around 10 years, the spread is 1 to 2 ticks. At longer maturities, the spread is 4 to 10 ticks. These spreads are a bit tighter than those observed in 2003 as discussed by Sack and Elsasser (2004).

3.1 Discount Function and Zero-Coupon Yields: Nominal and Real

The starting point for pricing any nominal fixed-income asset is the *nominal discount function*, or the price of a nominal zero-coupon bond. This represents the value today to an investor of a \$1 nominal payment n years hence. We denote this as $P_t^{nom}(n)$. The continuously compounded yield on this nominal zero-coupon bond can be written as

$$y_t^{nom}(n) = -\ln(P_t^{nom}(n))/n, \tag{1}$$

and conversely the zero-coupon bond price can be written in terms of the yield as

$$P_t^{nom}(n) = \exp(-y_t^{nom}(n)n). \tag{2}$$

Here and throughout, yields and coupon rates are expressed in percentage points; for example, 1 percent is written 0.01.

In the same way, we consider a *real discount function*, or the price of a real zero-coupon bond. We denote this as $P_t^{real}(n)$. This represents the value today to an investor of a \$$\frac{Q_{t+n}}{Q_t}$ payment n years hence where Q_t denotes the price index at time t.[5] The continuously compounded nominal yield on this bond is $\ln(\frac{Q_{t+n}}{Q_t P_t^{real}(n)})/n$. The continuously compounded real yield on this bond can be written as $y_t^{real}(n) = -\ln(P_t^{real}(n))/n$. Throughout this paper we discuss real bonds in terms of their real yields and nominal bonds in terms of their nominal yields.

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[5] In this discussion, we are abstracting from the indexation lag.

3.2 Par yields and forward rates.

There are a number of ways of expressing the information in the nominal yield curve in addition to continuously compounded zero-coupon yields. One way is to solve for the coupon rate which ensures that the price of the bond today will equal its maturity price. This is known as the par yield and is the market convention used to describe bond yields. Par yields are quoted with semiannual compounding because coupons on U.S. Treasury securities are paid twice per year. An alternative way of describing the yield curve is in terms of forward rates. We can solve for continuously-compounded instantaneous forward rates at all horizons. Or we can solve for m-year forward rates beginning n-years hence.[6]

All of these concepts apply to both TIPS and nominal securities, giving us two parallel sets of yields. In this paper, let $f_t^{nom}(n)$ and $f_t^{real}(n)$ denote the n-period ahead nominal and real instantaneous forward rates, and let $f_t^{nom}(n,m)$ and $f_t^{real}(n,m)$ denote the *m*-year nominal and real forward rates beginning *n* years hence, respectively. In addition, we will use $y_t^{p,nom}(n)$ and $y_t^{p,real}(n)$ to denote the *n*-year nominal and real par yields, respectively.

3.3 The Nelson-Siegel-Svensson yield curve

When fitting yield curves one faces a tradeoff between goodness of fit and smoothness of the curve. Spine-based nonparametric curves can fit the yields of individual securities arbitrarily closely, but at the cost of being quite jagged. The choice in this dimension depends on the purpose that the yield curve is intended to serve. For example, Cochrane

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[6] To further confuse matters, the m-year forward rates can be expressed as either zero-coupon forward rates or par forward rates.

and Piazzesi (2008) argue that the fourth and fifth principal components of the nominal yield curve—that explain a tiny share of the variation in yields—are nonetheless very useful for forecasting excess returns. If so, in forecasting excess returns, using a smoothed curve could lose the information in the fourth and fifth principal components. On the other hand, if the purpose of the yield curve is to understand its fundamental macroeconomic determinants, a spline-based curve may produce implausibly jagged yields and especially forward rates that will not be very useful in macroeconomic analysis. Our TIPS yield curve is designed primarily for macroeconomic interpretation and policy analysis. Thus, rather than fitting a spline-based curve, we impose some structure on the shape by imposing a parametric form that nonetheless fits the TIPS yields remarkably well. The benefit of the parametric approach is that it smoothes through the idiosyncratic movements in yields of individual securities and accurately represents the underlying shape of the (real) discount function.

The yield curves that we fit assume that the instantaneous forward rates (whether real or nominal) follow the functional form

$$f_t(n) = \beta_0 + \beta_1 \exp(-n/\tau_1) + \beta_2(n/\tau_1)\exp(-n/\tau_1) + \beta_3(n/\tau_2)\exp(-n/\tau_2). \qquad (3)$$

This structure was proposed by Svensson (1994) and is an extension of the functional form earlier used by Nelson and Siegel (1987). We refer to it as the NSS (Nelson-Siegel-Svensson) functional form. The original Nelson-Siegel functional form is a special case of (4) in which $\beta_3 = 0$. Integrating these forward rates gives us the corresponding zero-coupon yields:

$$y_t(n) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{n}{\tau_1})}{\frac{n}{\tau_1}} + \beta_2[\frac{1-\exp(-\frac{n}{\tau_1})}{\frac{n}{\tau_1}} - \exp(-\frac{n}{\tau_1})] + \beta_3[\frac{1-\exp(-\frac{n}{\tau_2})}{\frac{n}{\tau_2}} - \exp(-\frac{n}{\tau_2})]. \ (4)$$

The assumed functional form of the forward rates (and hence yields) has some intuitive characteristics. As discussed in more detail in Gürkaynak, Sack, and Wright (2007), it allows the forward rate curve the flexibility to start and end at estimated parameters, and to have a hump-shaped pattern in-between. The Nelson-Siegel functional form effectively allows for only one hump whereas the Svensson curve allows for two humps, which is important to capture convexity effects at longer horizons. The estimated parameters will determine the magnitude and location of these humps.

Given any candidate set of parameters, we can use (4) to solve for the nominal and real discount factors. With these discount factors, we can construct a predicted price for any Treasury security (nominal or real) with a given maturity date and coupon rate by considering it to be a bundle of zero-coupon securities, one corresponding to each payment on the security, with the value of each payment determined by the appropriate discount factor. We then estimate the nominal and TIPS yield curves by numerically choosing the set of parameters so as to minimize the weighted sum of squared deviations between actual and predicted prices. The weights are the inverse of the durations of each individual security.[7]

The ranges of maturities available for estimation over our sample are shown graphically in Figure 1, which takes the same form as a figure reported by Bliss (1996). The date is shown on the horizontal axis, the remaining maturity is shown on the vertical axis, and each outstanding TIPS security is represented by a dot showing its remaining maturity on that date. For example, a dot at a ten-year maturity in 2000 denotes a

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[7] Weighting price by inverse duration converts the pricing errors into yield fitting errors, to a first approximation. Fitting inverse-duration-weighted prices rather than yields is preferable because it is computationally much faster and delivers essentially the same yield curve.

security that is to mature ten years later, in 2010. The remaining maturity of that security will be traced out by a downward-sloping line, with the maturity shrinking as time passes.

As noted before, the Svensson curve allows for two humps, while the original Nelson-Siegel functional form allows for just one hump. The second hump is however not well identified unless we have enough long-term securities. For the nominal yield curve we used the restricted functional form up to 1980, but then we switched to the Svensson parameterization after that point because more long-term nominal Treasury securities became available to estimate the shape of more distant forward rates. Following similar reasoning, and given the maturities available as shown in Figure 1, for the TIPS yield curve we used the more restricted functional form up to the end of 2003, and we switch to the Svensson parameterization after that point as the issuance of 20-year TIPS helped the estimation.

TIPS with less than 18 months to maturity are dropped from the estimation of the TIPS yield curve, because the effect of the indexation lag makes the prices of these securities erratic. TIPS with remaining maturity between 18 and 24 months are downweighted linearly for smooth behavior of the short-end of the curve over time. All other TIPS are included in estimation of the TIPS yield curve.

3.4 Inflation Compensation

Having computed nominal and TIPS yields, it is straightforward to solve for rates of inflation compensation. These rates are defined as the inflation rates which, if realized, would leave an investor indifferent between holding a TIPS and a nominal Treasury security.

9

The formula is simplest for the continuously compounded zero-coupon inflation compensation rate:

$$\pi_t(n) = y_t^{nom}(n) - y_t^{real}(n).$$

It simply states that the cumulative amount of inflation needed to equalize the return on nominal and real zero-coupon securities, expressed as an annual inflation rate on a continuously-compounded basis, is given by the difference in the nominal and real yields. Similarly, the continuously compounded instantaneous forward inflation compensation rate is:

$$\pi_t^f(n) = f_t^{nom}(n) - f_t^{real}(n).$$

For par securities, the semi-annually compounded inflation compensation rate is given by:

$$\pi_t^p(n) = 2\left(\frac{1 + \dfrac{y_t^{p,nom}(n)}{2}}{1 + \dfrac{y_t^{p,real}(n)}{2}} - 1\right).$$

4. Yield curve results

Using the above methodology, we estimate the U.S. TIPS yield curve using daily data from January 1999 to the present (the data used in the paper go through October 21, 2008). Our underlying quotes on individual TIPS are kindly provided to us by Barclays Capital Markets.[8] The quotes are the averages of bid and ask prices.

As an example of the results, Figure 2 shows the estimated TIPS yield curve on June 1, 2005. The solid line is the continuously compounded par yield curve, the circles

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[8] We are not permitted to release the underlying data. However, the estimated yield curve is publicly available and regularly updated, as described in the text.

are the actual quotes on all outstanding coupon securities included in the estimation, and the crosses are the predicted yields for these issues.[9]

As can be seen, the yield curve generally does an impressive job fitting the entire cross-section of TIPS issues with a function of only six parameters. The success at fitting TIPS yields on this date is repeated throughout the sample. Figure 3 shows the average absolute yield prediction error in different maturity buckets over time. As can be seen, all of the errors are quite small over the entire sample. The largest fitting errors tend to be seen in the very shortest (2-5 year) and longest (20-30 year) maturity buckets, and even there the typical errors are only a few basis points.

Figure 4 returns to the specific date considered in Figure 3, only now showing the zero-coupon TIPS yield curve and the instantaneous forward rates at all horizons, as well as the corresponding measures for inflation compensation. In June 2005, the Federal Reserve was in the middle of a tightening cycle. The TIPS yield curve sloped up at that time, reflecting expectations for the continued removal of monetary policy accommodation and perhaps a real-interest-rate risk premium that was increasing in maturity. Because of convexity effects, the upward slope of the zero-coupon TIPS yield curve tapers off at long horizons and eventually turns down. Accordingly, the forward rates turn down earlier and much more sharply. Inflation compensation slopes down at short to intermediate maturities, perhaps reflecting expectations for a moderation in headline inflation, but then slopes up at intermediate and longer maturities, presumably owing in part to an inflation risk premium that is increasing in maturity.

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[9] That is, the crosses are consistent with the par yields shown in the line. They are not exactly on the par curve because the outstanding securities are not trading precisely at par.

The history of five- and ten-year zero-coupon nominal and TIPS yields is shown in Figure 5. In 1999, TIPS yields were high, reflecting in part the premium that investors demanded to induce them to hold these assets that were then quite illiquid. TIPS yields subsequently declined sharply, as liquidity improved and the FOMC eased monetary policy to combat the 2001 recession. The FOMC began to tighten monetary policy again in the middle of 2004. Five-year zero-coupon nominal and TIPS yields began to rise, as expected, but ten-year yields actually fell for a while, a phenomenon called the "conundrum" by former Fed Chairman Greenspan and discussed in Kim and Wright (2005) and Backus and Wright (2007),

The second half of 2007 and early 2008 was marked by the onset of turmoil in financial markets, and monetary policy was eased sharply to support economic growth. Accordingly, real rates plunged, with the five-year TIPS yield falling close to zero, and indeed shorter-term TIPS yields were negative; TIPS yields can and do fall below zero.[10] However, real yields rebounded starting later in 2008 as financial market conditions continued to deteriorate, even though monetary policy was eased further. This could reflect expectations for greater bond issuance caused by the "bailout" of financial institutions. Also, it could reflect the greater premium that investors demanded to hold relatively less liquid TIPS securities at a time of financial market stress.

The behaviors of five- and ten-year zero-coupon and instantaneous forward inflation compensation over the sample are shown in Figure 6. Inflation compensation was quite low in 1999, reflecting the poor liquidity of TIPS relative to nominal securities

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[10] The Treasury department has issued rules for how a TIPS auction would be conducted in a negative real rate environment. The coupon would be set to zero, and the TIPS would sell for below par value, at a price determined at auction. However, our fitted TIPS yields have never fallen below zero at the maturities at which the Treasury department conducts auctions, and there has never been a negative-yield TIPS auction.

(the counterpart of the high TIPS yields at that time). Breakeven inflation rates tended to drift higher through 2003, in part because of the improvement in TIPS liquidity. In addition, investors may have become more concerned about upside inflation risks by early 2004, in light of the updrift in realized inflation, the rebound in growth, and the continued accommodative stance of monetary policy.

At the very end of the sample period, five- and ten-year inflation compensation plummeted, presumably reflecting expectations for economic weakness to restrain inflation and also a further spike in the compensation that investors demand for the relatively low liquidity of TIPS. Distant-horizon forward measures of inflation compensation however increased somewhat, which could reflect a concern that the exceptional policy actions being undertaken in response to the credit crisis may lead to higher inflation in the long-run.

One inflation compensation measure that is of particular interest is the five-year forward five-year rate (also known as the five-year five-year forward breakeven inflation rate), because it has been explicitly mentioned by a number of Fed policymakers in their speeches. The behavior of this measure is often taken as a gauge of the Fed's inflation-fighting credibility. Policymakers seem to look to this measure to help judge whether near-term inflation pressures are working their way into longer-term expectations. The concern is that such leakage would create a more persistent inflation problem that would then be costly to reverse. If the Fed maintains its credibility, then the forward inflation compensation measure should be relatively unresponsive to information about the near-term outlook.[11] This measure, which can be derived from our yield curves as a par

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[11] The view that forward inflation compensation rates cannot be read as pure inflation expectations because of the presence of an inflation risk premium and a liquidity premium provides some room for the measure

forward rate over a five-year horizon, is shown in Figure 7. It shares many of the same properties of the instantaneous forward rates discussed above.

The yield curve estimated above is intended to be flexible enough to capture the general shape of TIPS yields while smoothing through some of the factors that might affect individual securities. One such factor is the seasonality of CPI. [12] Because TIPS are indexed to non-seasonally adjusted CPI, the yield on an individual security will reflect the expected seasonal change in the index ratio between the quote date (or more specifically the settlement date) and the maturity date of the security. This has two effects; it causes some variation in yields over time as the seasonality of the quote date changes, and it causes some variation across individual securities. For example, TIPS securities that mature in April tend to have higher yields and lower breakeven rates than those maturing in January, because the CPI seasonal corresponding to the April maturity is much lower than that corresponding to the January maturity (reducing the price of the April TIPS and hence raising their yield). At longer maturities, the impact of the seasonal pattern gets amortized over a longer period and is negligible, but it can be important at shorter horizons. The NSS yield curve, however, smoothes through this variation across maturity months. This feature is desirable for our curve, as it is designed to extract information about macroeconomic expectations and risks.[13]

The appendix that accompanies this paper provides data on zero-coupon yields (continuously compounded), instantaneous forward rates (continuously compounded),

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to vary without raising concerns among policymakers. However, risks to the inflation outlook are also important and a large enough rise would be concerning to a central bank, regardless of whether it was driven by inflation expectations or investors' assessment of considerable upside inflation risks.

[12] Ejsing, Garcia and Werner (2007) study the seasonal effect in euro area inflation compensation and show that for very short maturities the magnitudes involved are non-trivial.

[13] One might design a different curve, or append this one with additional analysis, if the main purpose is to gauge the relative value of specific securities.

and par yields (coupon-equivalent) for TIPS securities at a range of maturities as well as the five-year forward par five-year TIPS yield (coupon-equivalent). The maturities included span from 5 to 20 years for the period through 2004, and from 2 to 20 years for the period since then (when we use the more flexible specification, as discussed above). Yields at maturities below 5 years are not reported before the end of 2004 because, as can be seen in Figure 1, this would at times have entailed extrapolating the TIPS yield curve outside of the range of maturities that were actually outstanding. Meanwhile, yields for maturities below 2 years are never reported because the shortest maturity TIPS are excluded from our smoothed yield curve on account of the effects of the indexation lag and seasonality in the CPI.

The data are daily and are available back to January 4, 1999. The appendix also includes estimates of the six parameters of the NSS TIPS yield curve and the zero-coupon, instantaneous forward and par rates of inflation compensation, and the five-year forward five-year rate of inflation compensation. This appendix is posted on the website http://www.federalreserve.gov/econresdata/researchdata.htm, using the mnemonics described in Table 1. The data will be updated regularly as a resource for academic researchers and financial market practitioners.

5. Inflation compensation and inflation expectations

Inflation compensation is a mechanical calculation of the level of inflation which, if realized, would give investors the same return on TIPS and nominal Treasury securities. It is often used as a measure of inflation expectations, but this is correct only if investors are risk-neutral and there is no liquidity premium. Our interpretation of movements in

inflation compensation has stressed the existence of factors other than inflation expectations. In this section, we provide some further analysis supporting the view that variation in the inflation risk premium and the liquidity premium are also part of the behavior of inflation compensation. Inflation compensation is inflation expectations *plus* an inflation risk premium *minus* a liquidity premium that investors demand to hold comparatively less liquid TIPS securities.

5.1 The volatility of forward inflation compensation rates

In this subsection we address whether distant-horizon forward rates of inflation compensation can be viewed simply as measures of the long-run expected level of inflation, or the implicit inflation target of the central bank. If a ten-year forward rate of inflation compensation really is the rational long-run expectation of inflation, then it should be a martingale. Otherwise, the expectation of the long-run expectation of inflation tomorrow would differ from the long-run expectation of inflation today, which is impossible under rational expectations by the law of iterated expectations.

More precisely, let $\pi_t^f(10)$ and $\pi_t^f(10-)$ denote the ten-year and ten-year less one day forward rates of inflation compensation.[14] If the forward rates of inflation compensation represent inflation expectations, then $E_t(\pi_{t+1}^f(10-) - \pi_t^f(10)) = 0$ and so $x_t = \pi_{t+1}^f(10-) - \pi_t^f(10)$ is a martingale difference sequence. This hypothesis can be tested by a variance ratio test. Table 2 shows the standard deviation of x_t and $\Sigma_{j=1}^k x_{t+j}$ where k is 22, 66, or 132, corresponding to one, three and six months, respectively. The

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[14] The ten-year less one day rate of inflation compensation can be obtained from our parametric yield curves, assuming 260 business days in a year.

standard deviation of a one-day change in forward inflation compensation is a bit less than 5 basis points. If x_t really is a martingale difference sequence, then the variance of $\Sigma_{j=1}^{k} x_{t+j}$ must be k times the variance of x_t. Table 2 reports variance ratio statistics testing this hypothesis. The test statistic is $z^*(q)$ in the notation of Lo and MacKinlay (1988), which means that it is the variance ratio statistic that allows for time-varying conditional heteroskedasticity. Under the martingale hypothesis, this test statistic has a standard normal asymptotic distribution. However, we see in Table 2 that the test rejects in the left tail, meaning that the volatility of longer-term changes in inflation compensation is too small relative to the volatility of daily changes for inflation compensation to be a martingale.[15] This in turn means that there is some tendency to mean reversion in forward inflation compensation; when it is high, it subsequently falls, and *vice-versa*.

Stock and Watson (2007) find that an unobserved components model with stochastic volatility provides good forecasts for inflation. The model is a univariate model in which inflation is the sum of a martingale permanent component and a martingale difference sequence transitory component. The variance of the innovations to both components is allowed to be time-varying. At any point in time, the forecast of future inflation at any horizon is equal to the estimated permanent component. And the volatility of that forecast is the volatility of the permanent component. Stock and Watson find that the volatility of the permanent component was high in the early 1980s but has

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[15] It is well known that the distribution of this test statistic can be quite far from being standard normal in small sizes (Lo and Mackinlay, 1988 and Richardson and Stock, 1989). However, those papers find that the left-tail percentiles of the small sample distribution are above their asymptotic counterparts. Under these circumstances, the fact that we reject in the left tail is all the stronger evidence against the martingale hypothesis.

fallen since then. Estimating this model on headline CPI has averaged 20 basis points per quarter since the start of 1999. The standard deviation of one-day changes in ten-year inflation compensation is 5.6 basis points. So if we interpret ten-year inflation compensation as a long-run inflation measure and appeal to the martingale property that this implies, then the volatility of quarterly changes in long-run inflation expectations should be 45 basis points per quarter, which is more than twice as big as the time-series estimate. Again it appears that inflation compensation is too volatile at high frequency to represent inflation expectations alone.

5.2 Comparison to surveys of inflation expectations

Twice a year, in March and October, Blue Chip Economic Indicators collects long-range forecasts of CPI inflation five-to-ten years hence from a large number of professional forecasters. It seems worthwhile to compare the survey results with average five-to-ten year forward inflation compensation in each of the survey months. After all, at least at shorter horizons, surveys have been remarkably accurate predictors of future inflation (Ang, Bekaert and Wei, 2007).

Figure 7 shows the time series of the mean survey forecast, in addition to five-to-ten year forward inflation compensation. Inflation compensation has been far more volatile than survey expectations, and the two have no consistent relationship with each other. Since 2002, survey expectations have been consistently below inflation compensation, suggesting that the inflation risk premium (which pushes inflation

18

compensation up) now outweighs the TIPS liquidity premium (which pushes inflation compensation down). [16,17]

It is also of interest to study the association between inflation compensation and investors' uncertainty about inflation. Macroeconomic uncertainty is very hard to measure and we are not aware of any survey asking respondents for density forecasts at long horizons. But, the dispersion of long-horizon survey responses may serve as a crude proxy for uncertainty.[18] The Blue Chip surveys report a simple dispersion measure for their long-horizon survey questions, which is the difference between the average of the ten highest forecasts and the average of the ten lowest forecasts. Figure 8 plots the time series of this dispersion measure for CPI and five-to-ten year forward inflation compensation. There is a remarkable positive association between these two variables. In fact, the correlation between the survey dispersion and five-to-ten year forward inflation compensation in the survey months is 0.71, which seems reasonable if inflation compensation represents in part a risk premium that compensates investors for uncertainty about future inflation.

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[16] Our nominal yield curve does not include on-the-run issues and so our inflation compensation numbers are not distorted by the large and time-varying liquidity premia associated with these securities. Nonetheless, to the extent that TIPS are still less liquid than off-the-run nominal securities, inflation compensation may be pushed down by a premium to compensate investors for the differential liquidity of nominal and TIPS securities.

[17] This pattern has implications for the cost of TIPS issuance. In the early years of the TIPS program, TIPS were more expensive than nominal Treasury securities in terms of their expected servicing cost to Treasury, as argued by Sack and Elsasser (2004). However, with breakevens moving above expected inflation, the relative costs have been reversed for securities issued more recently, as pointed out by Roush (2007).

[18] Gürkaynak and Wolfers (2007) show that for several macroeconomic data releases the uncertainty and heterogeneity of beliefs are positively correlated but that these correlations are not very high. The dispersion of survey answers is therefore likely to capture some element of underlying uncertainty, but is an imperfect measure.

6. Decomposing inflation compensation

The previous section showed evidence that inflation compensation is not a pure measure of inflation expectations. In this section, we attempt to decompose inflation compensation into its components: inflation expectations, the liquidity premium, and the inflation risk premium.

6.1 The liquidity premium

We first estimate the TIPS liquidity premium, by regressing inflation compensation on measures of liquidity, following authors such as Chen, Lesmond, and Wei (2007) who estimated the effects of liquidity on corporate yield spreads in this way.[19] We use two measures of the relative liquidity of the nominal and index-linked bond markets.[20] The first is the trading volume among primary dealers in TIPS, expressed as a share of total Treasury trading volume, from the FR-2004 survey of primary dealers conducted by the Federal Reserve Bank of New York. This share rose from about ½ percent in 1999 to about 2 percent in 2006, as the TIPS market developed, but it remains small.[21] The second is the spread between Resolution Funding Corporation (Refcorp) strips and Treasury strips. Refcorp issued bonds to finance the resolution of the Savings and Loan crisis. These bonds are guaranteed by the Treasury and so have the same credit risk as Treasury securities, but they are considerably less liquid than Treasuries.[23] Thus, the

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[19] Chen, Lesmond and Wei (2007) control for measures of default risk. But since both the bonds we are considering are issued by the Treasury, there is no need to control for default risk.

[20] Ideally, we would use bid-ask spreads in the TIPS market, but unfortunately we do not have access to such data.

[21] TIPS constitute about 10 percent of Treasury issues outstanding, so their share in trading volume is well below their share in total supply of Treasury securities.

[23] To be precise, they have principal payments that are fully collateralized by non-marketable Treasury securities and coupon payments that are explicitly guaranteed by the Treasury. Also, they have the same tax treatment as Treasuries (subject to Federal tax and exempt from state and local tax).

spread between Refcorp and Treasury strips is a very direct indicator of the liquidity premium in the Treasury market, and has been used as such by Longstaff (2004). Of course, this measures the liquidity premium that investors demand to hold Refcorp bonds rather than nominal Treasury securities, which need not necessarily be the same as the liquidity premium that they demand to hold TIPS. Nonetheless, it seems reasonable to suppose that the Refcorp spread and TIPS liquidity premium should be highly correlated, as authors such as Chordia, Sarkar, and Subrahmanyam (2005) have argued that there is considerable commonality in liquidity premia, even across quite different markets (including stock and bond markets). Note that while our first measure (relative trading volumes) captures a physical measure of trading intensity, the second measure captures possibly time-varying prices of illiquidity as well.

We regressed five- and ten-year inflation compensation on these two liquidity measures jointly. The regression results are shown in Table 3; both are significant with the expected sign at both the five- and ten-year horizon. Rising TIPS volumes boost inflation compensation while an increase in the Refcorp spread lowers inflation compensation. The fitted values from this regression represent our measure of the time-varying effect of liquidity on inflation compensation. It does not, however, identify the level of the liquidity effect. We normalize this to zero in April 2005, meaning that we are measuring the liquidity premium *relative* to its value at that time (which was the period when TIPS liquidity premium was estimated to be the lowest in our sample). The estimated liquidity premium on five- and ten-year TIPS is shown in Figure 9. In this figure, the sign of the liquidity premium has been flipped so that it measures the extra

yield that investors demand to hold TIPS rather than nominal securities: a high liquidity premium drives inflation compensation down.

The estimated liquidity premium in yield terms is larger at the five- than at the ten-year horizon. The liquidity premium was high in the early years of the TIPS program, but fell fairly steadily between 1999 and 2005. During the recent period of financial market turmoil, the liquidity premium rose considerably, and it soared in September 2008, reaching about the same level as in 1999 when the TIPS market was small and still relatively new.

5.2 Extracting inflation expectations

We finally aim to construct a series of inflation expectations purged, to the extent possible, of liquidity and risk effects. This is a challenging task given the short sample period, but we attempt to do so using a state-space model in which survey expectations are treated as noisy measures of inflation expectations.

Concretely, we take the inflation compensation series at either the five- or ten-year horizon, adjusted by the liquidity premium as estimated in subsection 5.1, π_t^{ADJ}, and assume that this represents the sum of inflation expectations and an inflation risk premium:

$$\pi_t^{ADJ} = \pi_t^{EXP} + \pi_t^{RP} \tag{5}$$

Both the Survey of Professional Forecasters (SPF) and Blue Chip periodically ask respondents to forecast inflation over the next five and ten years. We assume that on the days of Survey of Professional Forecasters (SPF) surveys[24], the survey expectation of

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[24] The SPF is conducted around the start of each February, May, August and November. We treat the first business day of each month as the survey date. Likewise, the long-horizon Blue Chip surveys are

inflation at the five- or ten year horizon, π_t^{SPF}, can be written as a noisy measure of latent inflation expectations:

$$\pi_t^{SPF} = \pi_t^{EXP} + u_t^{SPF} \tag{6}$$

where u_t^{SPF} is iid measurement error. Likewise, on the days of long-horizon Blue Chip surveys, the survey expectation of inflation, π_t^{BC}, is assumed to be

$$\pi_t^{BC} = \pi_t^{EXP} + u_t^{BC} \tag{7}$$

where u_t^{BC} is iid measurement error with a separate variance. The combination of equations (5), (6) and (7) gives us the measurement equations for a system in state space form where $(\pi_t^{EXP}, \pi_t^{RP})'$ is the state vector. On most days, the survey expectations are treated as missing data. The transition equation assumes that long-run inflation expectations can be approximated by a random walk (motivated by the model of Stock and Watson (2007)) while the inflation risk premium is an AR(1), so that the transition equation is

$$\begin{pmatrix} \pi_t^{EXP} \\ \pi_t^{RP} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \pi_{t-1}^{EXP} \\ \pi_{t-1}^{RP} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} \tag{8}$$

where v_{1t} and v_{2t} are iid, mutually uncorrelated random variables with mean zero and variances σ_1^2 and σ_2^2, respectively. We set σ_1^2 to its estimated average value since 1999 from fitting the model of Stock and Watson to actual CPI inflation data. The Kalman filter can then be used to estimate the remaining model parameters by maximum likelihood, and smoothed estimates of inflation expectations can be extracted.[25]

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conducted around the start of each March and October and we treat the first business day of each month as the survey date.

[25] The inflation risk premium can also be extracted, though this is only identified up to a constant because the liquidity premium has only been identified up to a constant. Meanwhile, the assumption that the surveys measure inflation expectations correctly on average identifies the level of inflation expectations.

Figure 10 shows the five- and ten-year inflation expectations obtained in this way. Our estimates of inflation expectations generally moved in a fairly narrow range over this period. Inflation expectations were low at the start of the sample, which may partly owe to a difficulty in fully adjusting for the effects of poor liquidity at that time. Inflation expectations fell in 2003, at the time of the "deflation scare," before rebounding as the economy grew. Expected inflation also rose during most of the recent period of financial market turmoil, which is consistent with a view that the FOMC was focusing on supporting growth at the expense of its inflation objective. However, at the very end of the sample, our measure of inflation expectations fell sharply. Extracting macroeconomic expectations implicit in asset prices is particularly challenging in the fall of 2008, but this could be because agents expect a severe and long-lasting recession to restrain aggregate demand and inflation going forward—a view also echoed by policymakers.

7. Conclusion

In this paper we have estimated the U.S. TIPS yield curve using an approach that is simple and parsimonious. The methodology is quite effective at capturing the general shape of the yield curve while smoothing through idiosyncratic variation in the yields of individual inflation-protected securities. The estimated yield curve can be expressed in a variety of ways, including zero-coupon yields, par yields, and forward rates. And it can be compared to the corresponding nominal yield curve to obtain estimates of inflation compensation.

Having the real yield curve should provide tremendous benefits to our efforts to better understand the behavior of nominal yields. It allows us to parse nominal yields and

forward rates into their real rate component and their inflation compensation component. These two components may behave quite differently, in which case simply looking at a nominal yield might mask important information. Inflation compensation is a useful measure, because it is the only high frequency measure of market's concerns about inflation. Nonetheless, we argue that it embodies non-trivial and time-varying liquidity and inflation risk premia. Thus research on understanding not only the mean but the distribution of perceived future inflation outcomes, and the prices of risk associated with these is an important part of understanding the behavior of inflation compensation and the nominal yield curve.   We have taken a step in that direction by showing one way to decompose inflation expectations into its inflation expectations, inflation risk premium and liquidity premium components.

We hope that our TIPS yield curve will be useful to researchers in further work that combines macroeconomics and finance. It is to this end that we have made the full dataset available to be downloaded on the web. These data will be updated periodically.

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Table 1: Description of the series in the data appendix

Series	Compounding Convention	Mnemonics	Maturities Reported (max)
TIPS yields			
Zero-coupon	Continuously Comp.	TIPSYXX	All integers 2-20
Par	Coupon-Equivalent	TIPSPYXX	All integers 2-20
Instantaneous forward	Continuously Comp.	TIPSFXX	All integers 2-20
One-year forward	Coupon-Equivalent	TIPS1FXX	4, and 9
Five-to-ten-year forward	Coupon-Equvalent	TIPS5F5	
Parameters	N/A	BETA0 to TAU2	N/A
Inflation compensation			
Zero-coupon	Coupon-Equivalent	BKEVENYXX	All integers 2-20
Par	Continuously Comp.	BKEVENXX	All integers 2-20
Instantaneous forward	Coupon-Equivalent	BKEVENFXX	All integers 2-20
One-year forward	Coupon-Equivalent	BKEVEN1FXX	4, and 9
Five-to-ten-year forward		BKEVEN5F5	

Notes: XX in each case denotes the maturity in years. For example, TIPSY10 denotes the ten-year zero-coupon yield. The one-year forward rates XX years hence denote the one-year forward rates *beginning* XX years hence. For example, TIPS1F09 is the one-year forward rate from nine to ten years hence. The parameters are labeled BETA0, BETA1, BETA2, BETA3, TAU1, and TAU2, corresponding to the equations in the text. Note that the parameters BETA3 and TAU2 are restricted to zero in the earlier part of the sample, as discussed in the text.

Table 2: Volatility of changes in ten-year forward inflation compensation at selected horizons

| Horizons	Standard Deviation (Basis Points)	Variance Ratio Statistic
One day	5.6	
One month	18.8	-3.67[**]
Three months	26.7	-2.86[**]
Six months	32.6	-2.35[**]

Notes: This table shows the standard deviation of one-day and one-, three- and six-month cumulative values of $x_t = \pi^f_{t+1}(10-) - \pi^f_t(10)$. They are computed assuming 22 days per month. The variance ratio statistic is the heteroskedasticity robust test statistic of Lo and MacKinlay (1988) and has a standard normal asymptotic distribution. *, ** and *** denote significance at the 10, 5 and 1 percent significance levels respectively. The number of daily observations is 2,449.

Table 3: Regression Results for Estimating Liquidity Premia in Five- and Ten-Year Inflation Compensation.

Predictor	Five-year	Ten-year
TIPS Relative Volume	0.51	0.40
	(0.09)	(0.06)
Refcorp Spread	-1.67	-0.78
	(0.64)	(0.34)
R-squared	0.422	0.423

This table reports the results from regressions of five- and ten-year inflation compensation (in percentage points) onto the TIPS volume, as a share of total Treasury Primary Dealer trading volume (in percentage points) and the spread of twenty-year Refcorp strips over their Treasury counterparts (in percentage points). Newey-West standard errors with a lag truncation parameter of 20 are shown in parentheses. The number of observations is 2,450.
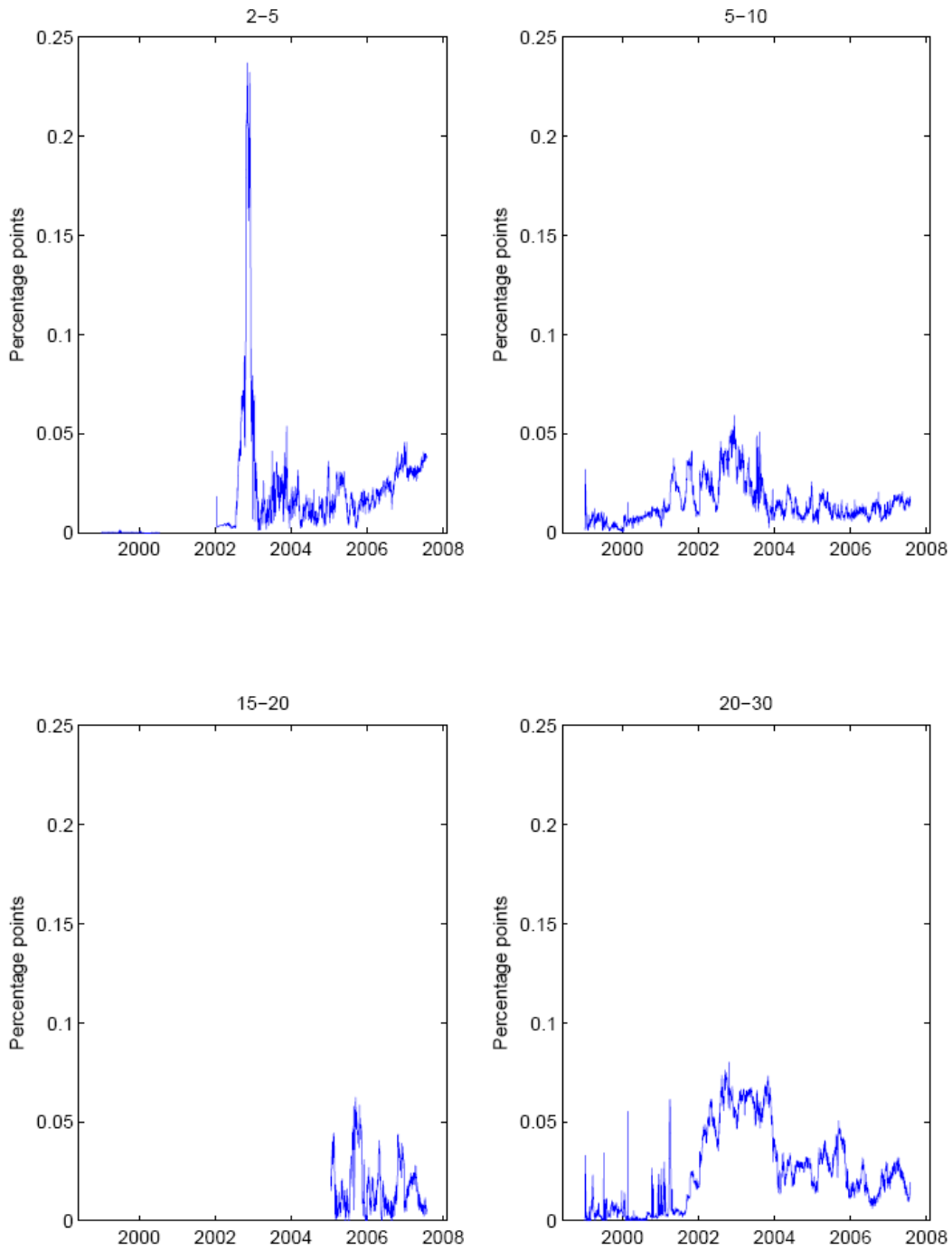
Figure 1: Outstanding TIPS Securities
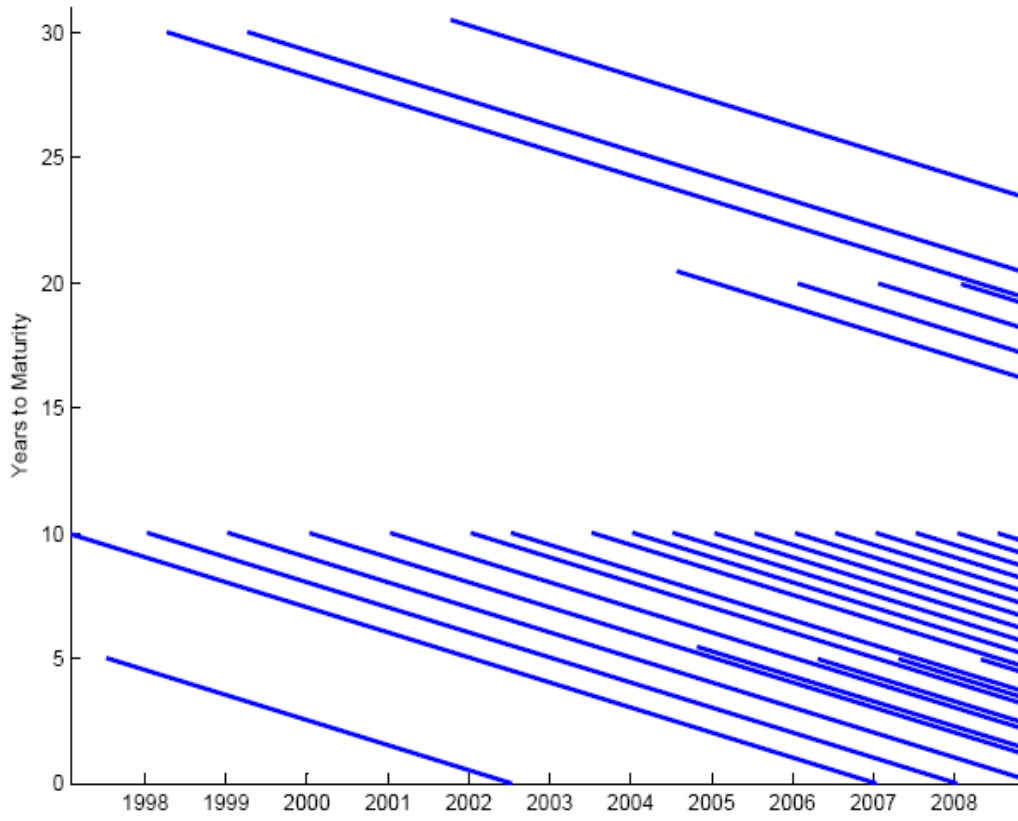
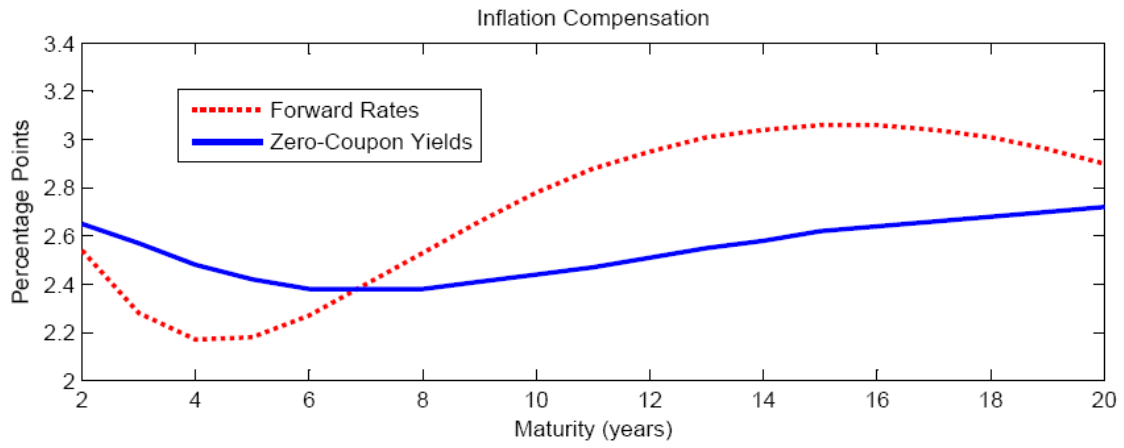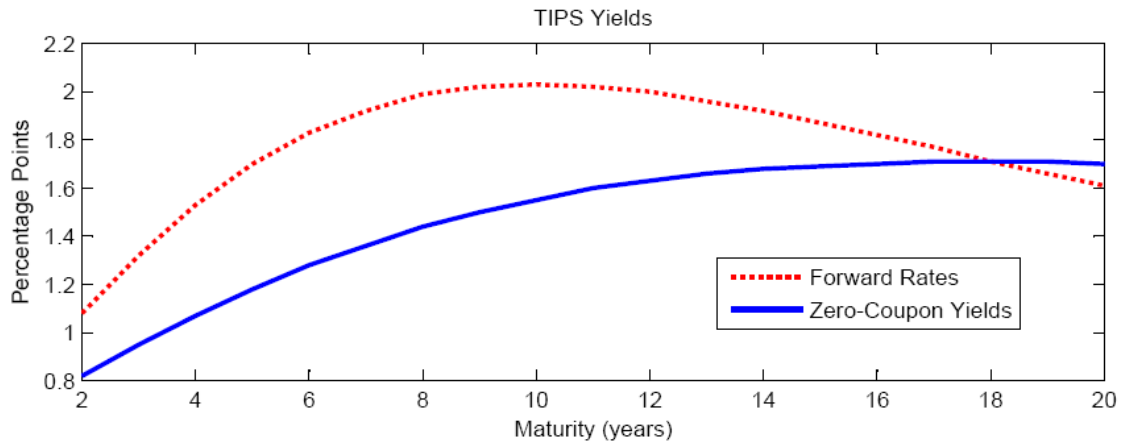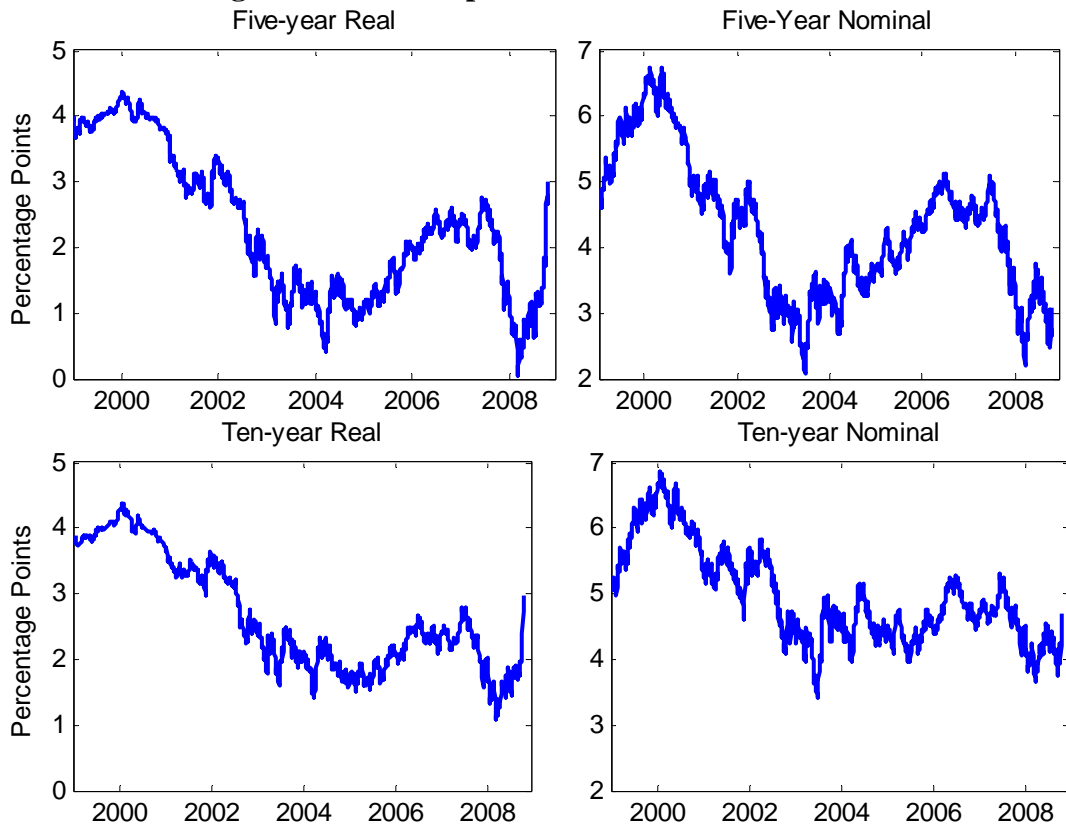**Figure 2: Par TIPS Yield Curve on June 1, 2005**
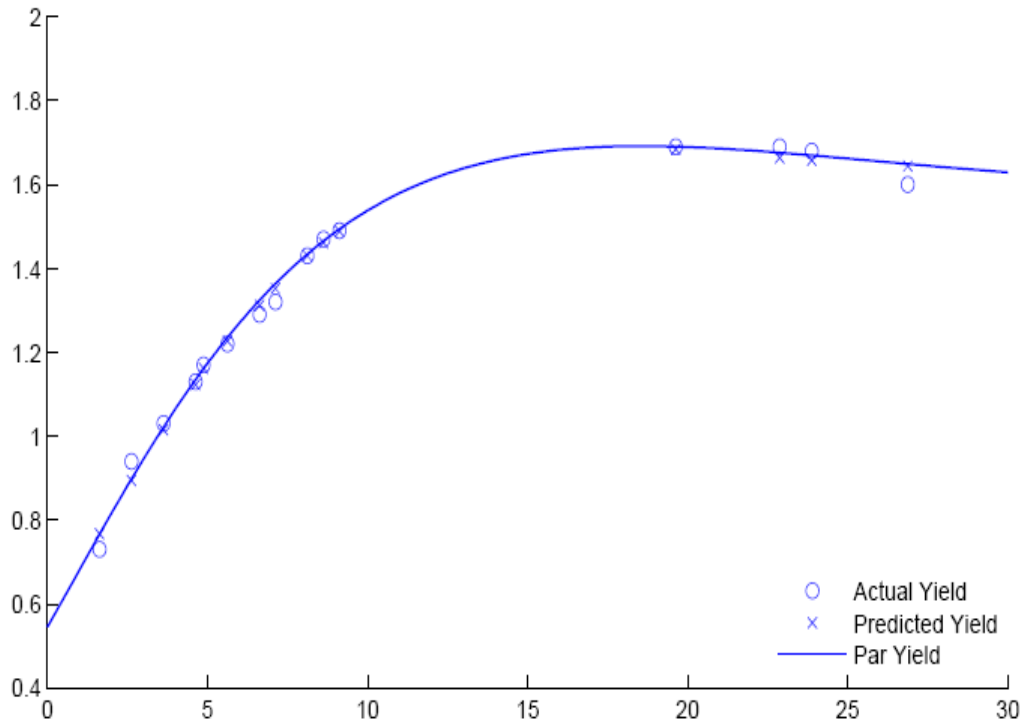
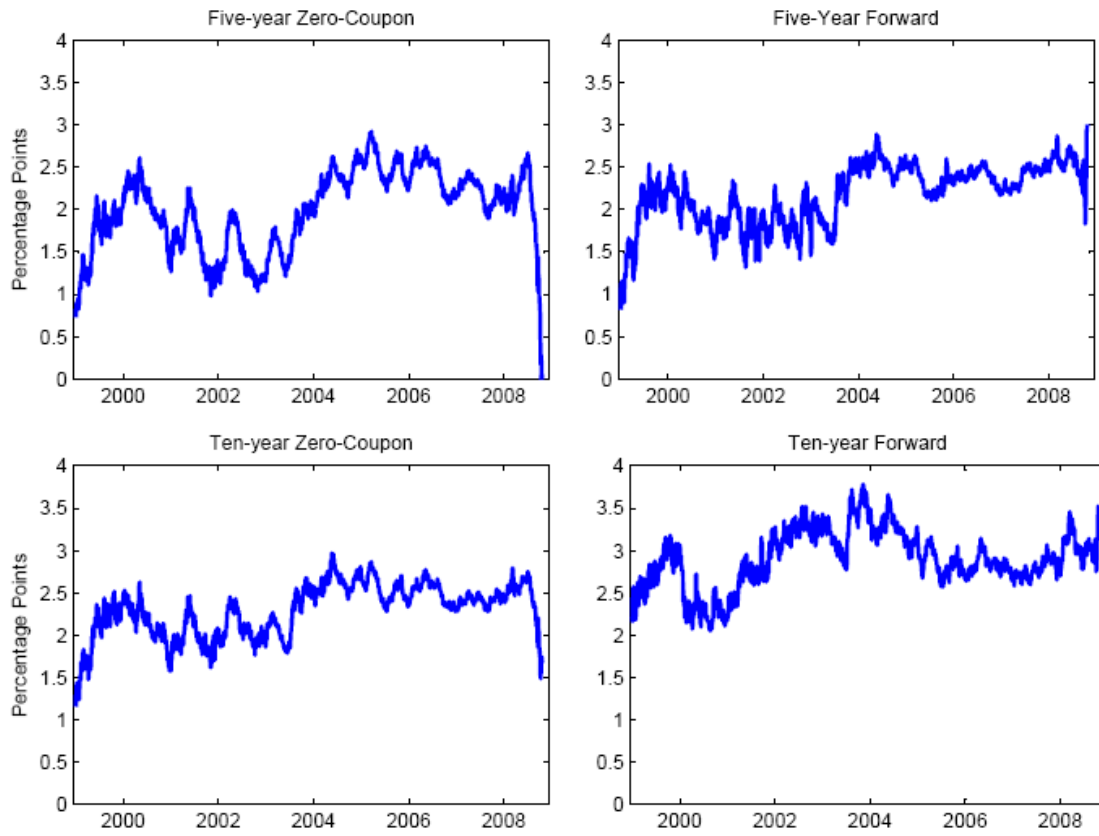**Figure 3: Average Absolute Yield Prediction Errors by Indicated Maturity Bin**

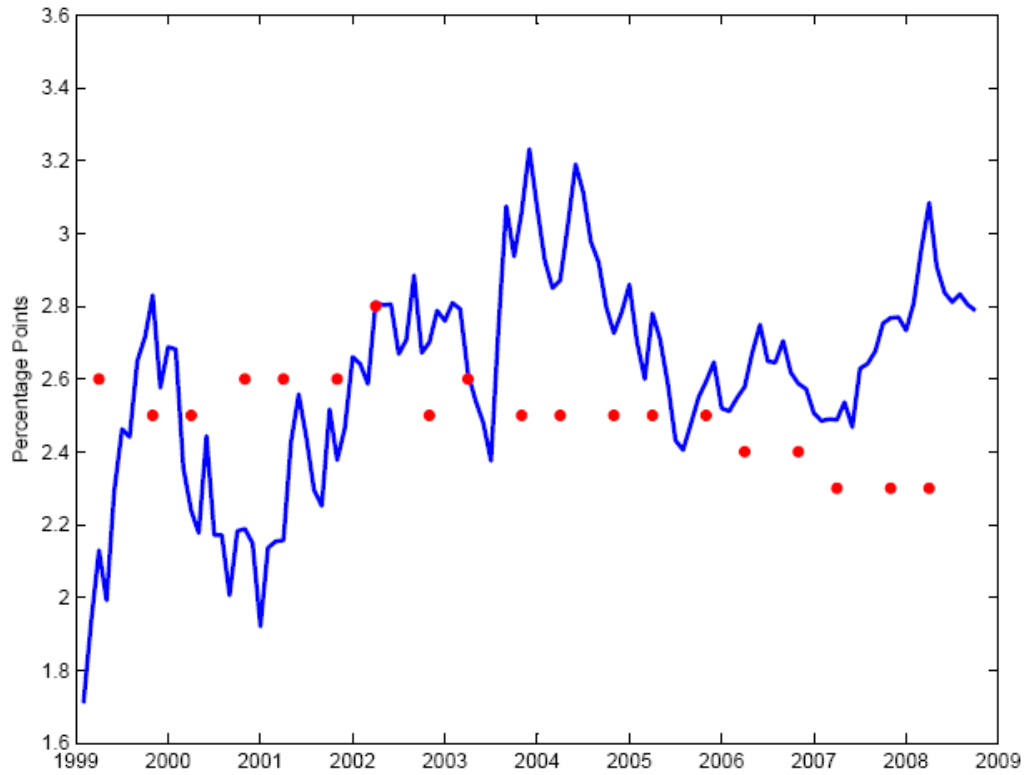**Figure 4: Zero-Coupon and Forward Rates on June 1, 2005**

Figure 5: Zero Coupon Yields: TIPS and Nominal
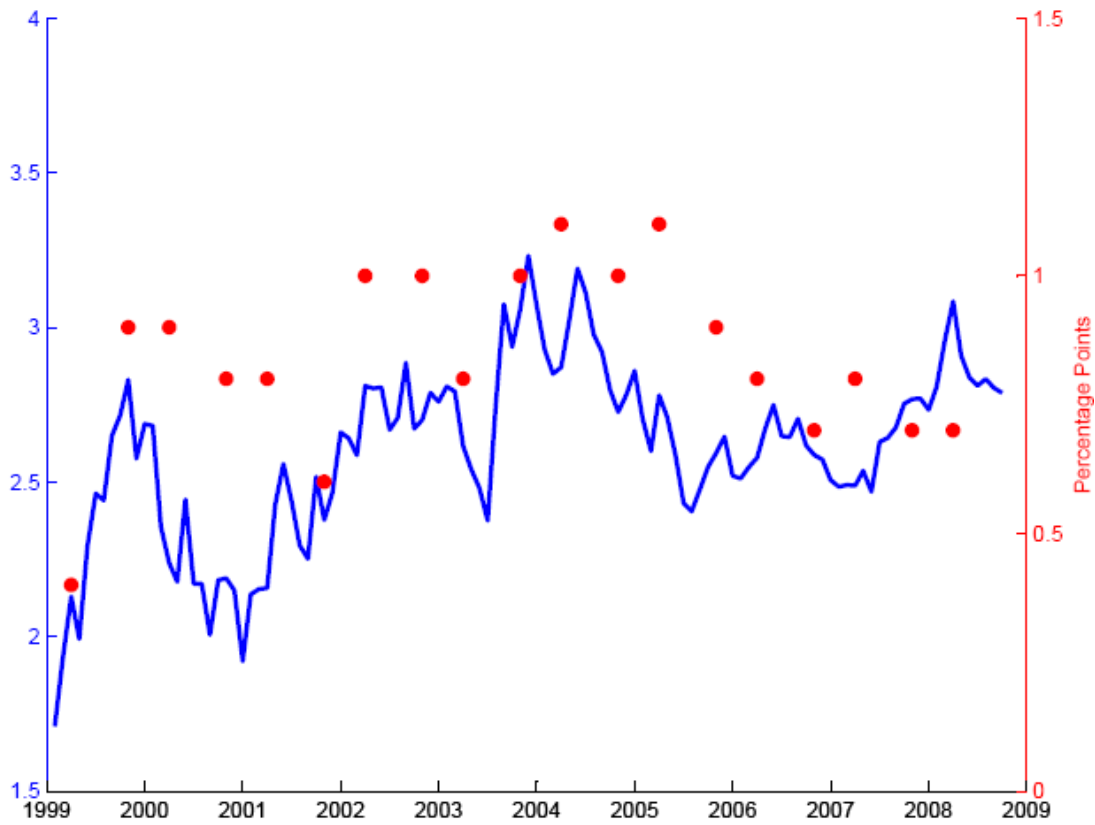
Figure 6: Inflation Compensation

Figure 7: Five-to-ten Year Forward Inflation Compensation & Blue Chip Forecasts


Notes: The solid line gives forward par inflation compensation. The dots are the Blue Chip survey inflation expectations.

Figure 8: Five-to-Ten Year Forward Inflation Compensation and Blue Chip Forecast Dispersion


Notes: The solid line gives forward par inflation compensation. The dots are the dispersion of forecasts from the Blue Chip survey.

Figure 9: Estimated TIPS liquidity premium
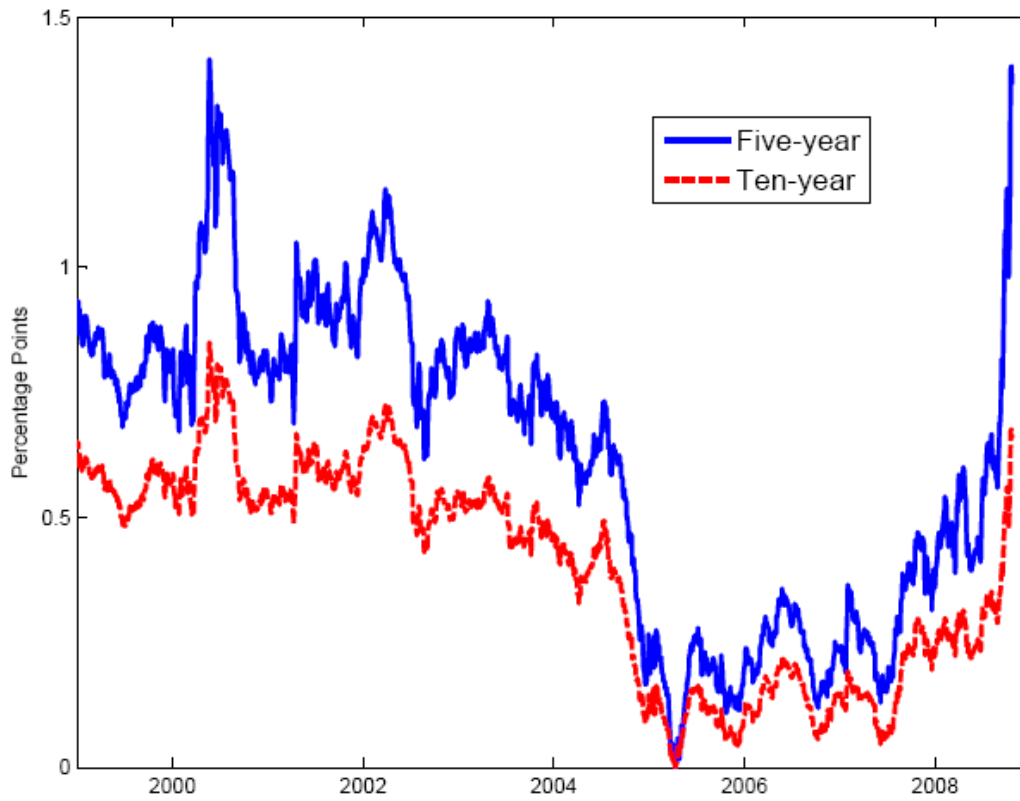
Notes: Regression-based estimates of the liquidity premium for holding TIPS relative to that for nominal securities, constructed as described in the text, normalized to zero in April 2005. The regression coefficient estimates are shown in Table 4.

Figure 10: Estimated Inflation Expectations
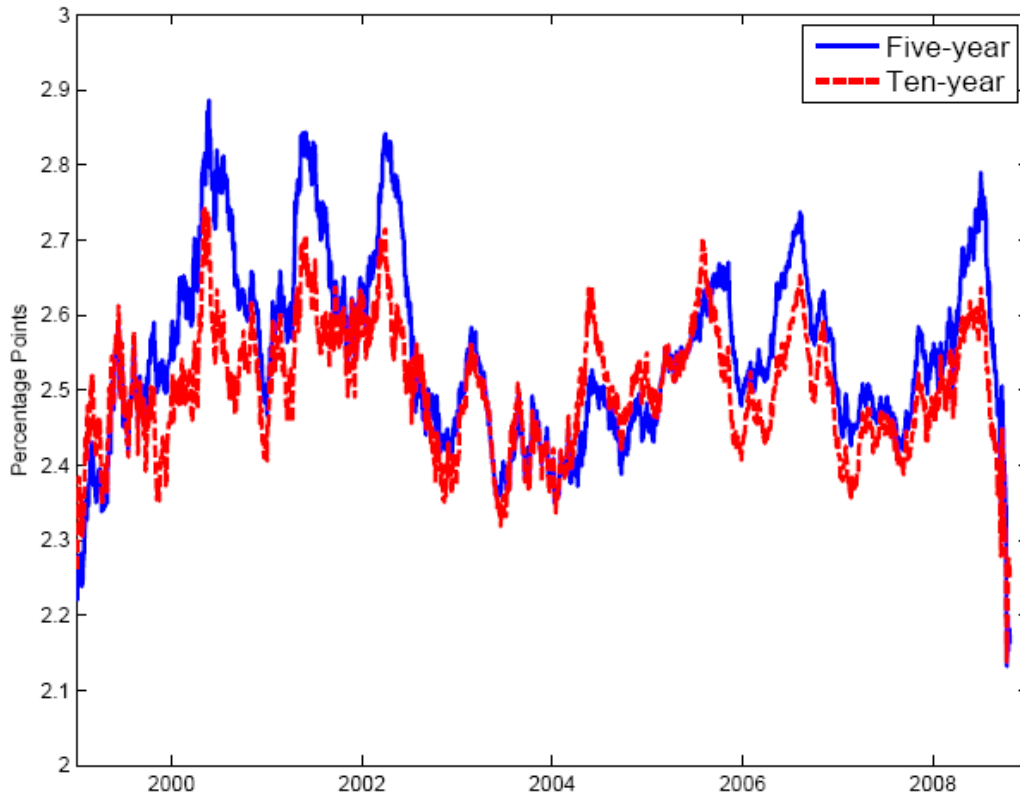
Notes: Estimates of inflation expectations at five- and ten-year horizons obtained from the Kalman smoother applied to the inflation compensation series adjusted for liquidity effects.